

Simulating a More Efficient Bike Sharing System

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Many cities operate bike sharing systems in which riders rent a bike at one station and return it a short time later to another station. Because almost all trips are one way, the supply of bikes and docks at some stations eventually becomes unbalanced with demand, preventing riders from either renting or returning a bike. Consequently, bike sharing system operators spend time and resources (including fossil fuels) moving bikes from stations with an excess of bikes to those with shortages. This article uses an animated discrete-event simulation model to investigate the possibility of changing the system's initial conditions to reduce shortage events. Results based on data from San Francisco's bike sharing system show that selectively adding just 3-4% to the total number of docks and bikes in the system, at just a handful of stations, can lower the number of dock and bike shortages by 30%, significantly reducing the amount of rebalancing required by the operator.

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I. INTRODUCTION

Urban bike sharing systems (BSS) encourage people to rent bikes in one location and return them a short time later at another location, facilitating the "last mile" of a commuter's trip as well as giving a healthy, green transport option for tourists and others who wish to move around the city. Such systems have existed in one form or another since first appearing in Amsterdam in the mid-1960s (DeMaio, 2009). Initially taking root in European cities, bike sharing systems over the past decade have grown rapidly around the globe in both number and sophistication. In fact, the total number of shared bikes worldwide rose from just a few thousand in a handful of cities in 2002 to nearly one million bikes in 855 cities by 2014, with more than three fourths of the bikes operating in China

(Richter, 2015). Furthermore, current systems use a variety of technologies including credit card payment, electronically-controlled docks accessed with fobs, on-board GPS devices to track bikes, and telecommunication systems that allow real-time bike and dock availability information to be accessible to users with smart phones (DeMaio, 2009).

In addition to lowering expenses, improving health, and providing more transportation options for participants, the purported benefits of bike sharing systems include reducing congestion and auto emissions. In practice, however, it's unclear whether or not such systems shift a meaningful percentage of single-occupant car drivers out of their cars onto bikes (Fishman et al., 2013). Another major issue that diminishes the environmental and financial benefits of a BSS is that, over time, the distribution of available

bikes becomes unbalanced with the demand for bikes, requiring BSS operators to periodically reposition (or rebalance) the bikes, typically via fossil-fueled trucks. To mitigate the environmental impact and financial cost of rebalancing, it may be possible for BSS operators to make relatively small changes in the system's initial conditions (the number of docks at each station and the number of bikes placed at each station at the start of the day) that will reduce the number of bike and dock shortages faced by riders.

The rest of this paper is organized as follows. Section 2 reviews related literature, while Section 3 discusses the BSS operating in San Francisco and some of its key data. Section 4 presents an animated discrete-event simulation model of San Francisco's system, while Section 5 discusses key performance measures for the system and how the model was validated. Section 6 describes experiments performed with the model to identify improved initial conditions. Finally, Section 7 provides conclusions and points to possible areas of future work.

II. RELATED LITERATURE

The literature on bike sharing differentiates between two modes of rebalancing: static and dynamic. In the static case, no user activity is assumed to be taking place during rebalancing. This is nearly true in most BSS bikes late at night, when bikes are repositioned in order to get the system ready for the next day. In the dynamic case, bikes are rebalanced multiple times throughout the day while the system's inventory of bikes is fluctuating. Although there appears to be more published work regarding the static case than the dynamic case, BSS operators in practice employ both static and dynamic strategies to rebalance their systems, as they are complementary in nature.

In the static setting, Benchimol et al. (2011) and Erdoğan et al. (2015) presented

heuristic and exact algorithms, respectively, for routing a *single* vehicle around a BSS at lowest cost, picking up and dropping off bikes at various stations to better prepare the system for the next day's demand. Going an important step further, Chemla et al. (2013), Rainer-Harbach et al. (2013), Raviv et al. (2013) and Dell'Amico et al. (2014) have all proposed heuristics to find the best routes for *multiple* vehicle to follow while determining how many bikes to pick up and drop off at each station along the vehicles' routes. In the same setting, Di Gaspero et al. (2013) presented a hybrid metaheuristic method in which the vehicle routing variables are handled by ant colony optimization (to minimize work done by the vehicles), while operational variables indicating the number of bikes to load or unload at each station are handled by constraint programming (to minimize deviations from the target number of bikes at each station).

In the dynamic case, Contardo et al. (2012) tackled the multiple vehicle problem with mathematical programming formulations that schedule vehicles to pick up and drop off bikes at stations while minimizing unmet demand, defined as the number of customers who try to rent a bike at an empty station or return one to a full station. They used decomposition techniques on their arc-flow formulation to find feasible solutions quickly. Schuijbroek et al. (2013) combined the determination of bounded service level requirements at each station, based on a finite capacity queuing system view of each station's demand, with the design of near-optimal vehicle routes to rebalance the bikes in any given period for which demand is stationary. Their cost minimizing heuristic decomposed the multiple vehicle problem into a set of single-vehicle problems that can be solved quickly. Kloimüller et al. (2014) adapted heuristics developed for the static case with the goal of finding efficient routes for multiple vehicles so that the BSS is brought into a

balanced state and is able to fulfill user demands as much as possible. However, while reducing operator costs and unsatisfied demand, all of these operator-based approaches in both the static and dynamic cases have relied on fuel-burning vehicles for rebalancing and incur the associated environmental costs.

Other researchers have turned to computer simulation to model the behavior of resource sharing systems and suggest improved rebalancing strategies. For example, Kek et al. (2005) employed discrete event simulation to test different relocation techniques in car sharing systems, one of which might allow for a 10% decrease in car parking lots and 25% less staff. Ji et al. (2014) used Monte Carlo simulation to evaluate the reliability of a proposed e-bike sharing system under various scenarios involving different numbers of batteries and e-bikes in the system; however, in their small, two-station pilot system, rebalancing was not considered since it only allows riders to make round trips. Caggiani and Ottomanelli (2013) simulated bike rebalancing in the dynamic case, aiming to minimize vehicle repositioning costs for the BSS operator while maintaining a high level of user satisfaction, but used an aggregated, period-based model that does not portray the behavior of individual riders.

The work presented here employs a detailed discrete-event simulation model, in conjunction with an optimization routine, to investigate how changing the system's initial conditions could reduce bike and dock shortage events. Resetting the number of docks at a small number of stations would be a one-time adjustment to the BSS, while adjusting the number of bikes at a few stations would need to occur at the end of each day, similar to static rebalancing efforts. One advantage of our approach is that the changes we recommend are simple and need only be made at a small number of stations. Moreover, because decision makers tend to have an easier

time understanding animated simulation models than complex algorithmic-based approaches, our recommendations may be more likely to be implemented by the system operator.

III. SAN FRANCISCO'S BIKE SHARING SYSTEM

In the San Francisco Bay Area, a leading bike share company called Motivate (Simons, 2015) operates a BSS consisting of approximately 700 bikes circulating among 70 stations in 5 cities: San Francisco, Redwood City, Palo Alto, Mountain View and San Jose. However, since 91% of the bike sharing rides taken in the Bay Area occur on bikes moving among San Francisco's 35 stations, we confined our analysis to the San Francisco portion of the larger Bay Area system. The locations of San Francisco's stations, which began operating in August 2013, can be seen in Figure 1. At present, in the pilot phase of this BSS, all stations are located in the relatively flat, largely commercial northeast quadrant of the city.

In August 2015 the BSS operator made data files from its first two years of operation publicly available on the Internet (Bay Area Bike Share, 2015). We used data from the second year (September 2014-August 2015) in this project; Schuijbroek et al. (2013) give details on the format of these types of files. The "Trip" file contains 322,437 records of anonymized trips taken within San Francisco, where each trip record gives the bike number, the start day and time, end day and time, starting and ending station, and the rider type (annual or casual). From this file, we deduced several key characteristics of the system and its riders using the JMP software package (SAS Institute, 2013), including the arrival rates to each station by hour and rider type, and the transition probabilities between each pair of stations by rider type. Analysis revealed that 90% of all rides in San Francisco

occur on weekdays, so we further confined our study to weekday travel within San Francisco. We also noticed that 92% of all weekday trips taken in San Francisco were made by annual members – typically daily commuters making short trips to and from work – while the

remaining 8% were made by casual riders whose trips tend to be both longer and more variable than those taken by annual members (median and IQR are 16.5 and 20.6 minutes, respectively, for casual riders versus 8.1 and 5.6 minutes, respectively, for annual riders).

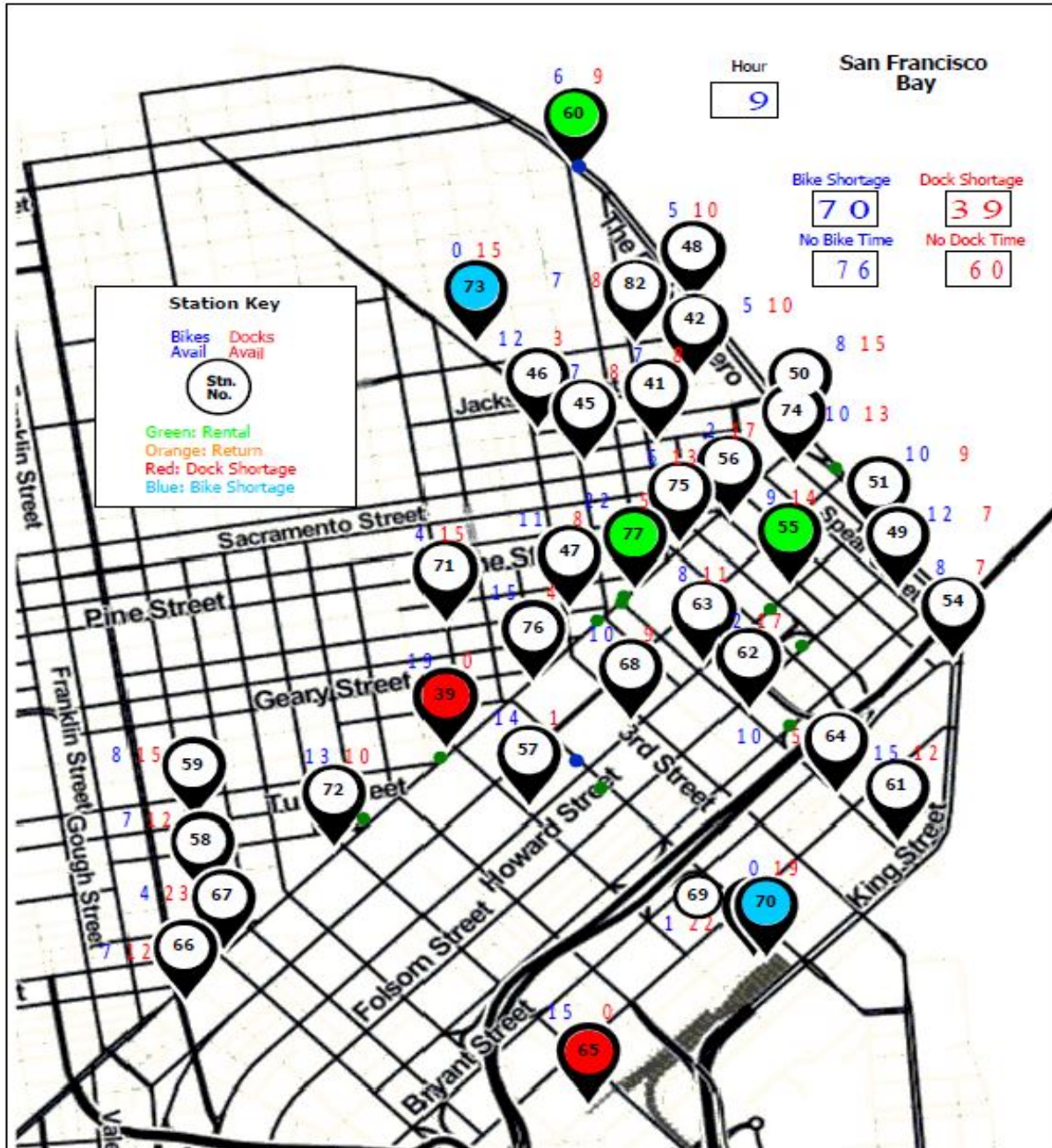


FIGURE 1. SNAPSHOT OF THE ANIMATED PORTION OF THE SFBS MODEL.

The “Status” file consists of more than 36 million records giving the number of bikes and docks available at each station every minute during the BSS’s second year of operation. Riders may access similar information in real time on their smart phones, e.g., to see if the station they’re approaching has an available bike or dock. We used these data to estimate the number of bikes available at the beginning of each hour of the day at each station, and the fractions of time stations had no available bikes and no available docks.

Finally, we gathered data in order to fit theoretical distributions for two variables using Arena’s Input Analyzer (Kelton et al., 2010). In particular, the time (in seconds) needed by a rider to remove a bike from its dock appears to follow the $10 + \text{Exponential}(15.1)$ distribution, while the time needed to lock a bike into a dock has the form of a $\text{Triangular}(2, 3, 6)$ distribution.

IV. A DISCRETE-EVENT DYNAMIC SIMULATION MODEL

To study the system, we built a detailed animated model, referred to here as the San Francisco Bike Share Simulation (SFBSS) model, using the Arena 14 software package (Kelton et al., 2010). The model’s major assumptions are the following.

1. Each hour’s bike demand follows a Poisson process whose mean comes from the Trip file.
2. Riders who find no bike available at their intended starting station wait a preset maximum amount of time (1 and 2 minutes for annual and casual riders, respectively) before exiting the system without having rented a bike and recording a bike shortage. Instead of sending unsatisfied riders to other stations to look for a bike, the model creates arrivals at stations in volumes that allow it to match up closely with

the observed number of completed rides. Real BSS are unable to track bike shortages, since they’re only alerted when bikes are removed from or returned to docks, so no actual data on bike shortages exists.

3. Since the trip duration distribution across all rides looks Lognormal, we assume the travel time distributions for all 1,225 origin-destination pairs for each rider type are Lognormal as well.
4. Riders who reach their destination station but find no docks available wait a preset maximum amount of time (1 and 2 minutes for annual and casual riders, respectively) before checking their smart phone app and riding to the nearest station with a dock available.

The logical flow of riders (the main entities) through the SFBSS model, seen in Figure 2, will now be described. First, the CreateArrivals submodel generates riders of each type using separate arrival streams, and places them at their starting stations based on historical percentages of trips originating at each station, which vary by hour of the day and type of rider. The model employs a generic bike station structure to greatly reduce the amount of code needed to accurately represent the system. Modeled riders possess an attribute called “Stn” which indicates their current station and allows them to index into the set of 35 stations.

Whenever riders reach a station, SFBSS first checks to see if they are renting or returning a bike. If renting, the model then determines if any bikes are available. If so, the rider rents a bike at this station and selects a destination station (PickNextStn submodel) based on historical transition probabilities. Riders are then assigned a travel time to their destination based on historical data for their type, and proceed toward the destination station. If the rider has taken the last bike at the rental station, the model sets a flag.

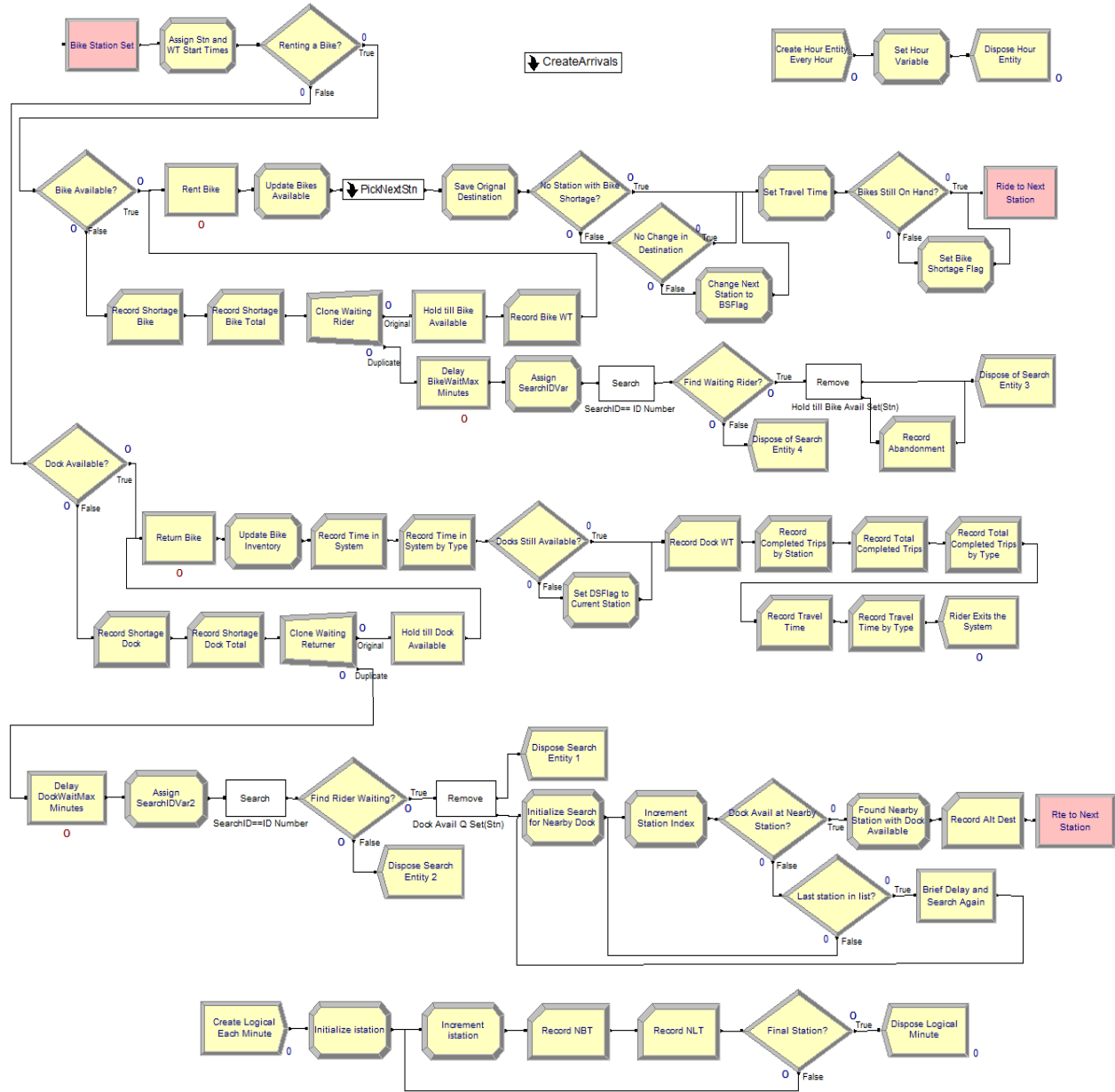


FIGURE 2. MAIN LOGICAL FLOW OF THE SFBSS MODEL.

If no bikes are available when a rider arrives at a station, the model records a bike shortage and the rider waits for a bike to become available. If a bike becomes available before the preset maximum amount of time has elapsed (see Assumption 2), the rider rents the bike and follows the logic described in the

preceding paragraph. Otherwise, the rider abandons the BSS without having made a trip. Riders reaching a station wanting to return their bike must check whether or not a dock is available. If one is available, they quickly return their bike to a dock and the model updates appropriate bike and dock counters.

The model then examines whether this station is out of docks, and if so, sets a flag to this station's number. Performance measures are updated before the rider exits the system.

When riders reach their destination station and find no docks available, SFBSS records a dock shortage and the rider then waits for a dock to open up at this station. If a dock becomes available before the preset maximum amount of time has passed (see Assumption 4), the rider returns the bike, and the model follows the logic of the preceding paragraph. Otherwise, riders find the station closest to their original destination with a free dock and ride toward it.

V. MODEL VALIDATION AND KEY PERFORMANCE MEASURES

Many system activities are animated on screen during model execution (see Figure 1), helping us verify that the SFBSS model behaves as intended, i.e., its code is valid. For example, the model shows annual and casual riders as green and blue circles, respectively. Station symbols change color depending on their status: green indicates that a rental is taking place, orange a return, red a dock shortage, and blue a bike shortage. Also, blue and red numbers next to each station indicate the number of currently available bikes and docks, respectively.

We validated the SFBSS model operationally by examining several system performance measures. While SFBSS tracks quite a few outputs, including many that are station-specific, we focus here on the major summary measures listed in Table 1 below: 1) the total time spent in the system, by rider type and overall; 2) the total number of rides completed, by rider type and overall; and 3) the fractions of time no bikes and no docks are available. Two related measures of interest

which the simulation model reports are the average number of bike and dock shortages per day. Interestingly, these undesirable events are not tracked by the BSS operator since its electronic data collection system only records when bikes are successfully rented at or returned to a station. (Dock shortages might be recorded if a rider takes the time to report it to the operator and request more time to find a dock.)

While San Francisco's BSS never completely shuts down, only 1.5% of all rides occur between midnight and 6 am, so we treated the simulated system as a terminating system that essentially restarts early each day. All performance measures from the model are averaged over 30 replications, each of which represents a 24-hour day.

The second and third columns of Table 1 show that SFBSS generates values that are close to those observed, in both absolute and percentage terms, for the time in system and number of completed trips. The table's last column gives 95% confidence intervals for the base case model means; the differences between the model and observed means are not statistically significant for any of the first six performance measures. However, the model does not match up well on the fractions of time with bike and dock shortages. This is due to the fact that the real BSS operator engages in periodic bike rebalancing, while the simulation model does not, leading to longer periods of simulated time in which stations may have either no bikes or docks available. Aside from these measures, the SFBSS model seems to credibly represent the real BSS in San Francisco, and provides a way to estimate the number of shortage events. It also provides a reasonably accurate tool with which to experiment to predict how the real system might react to possible modifications.

TABLE 1. KEY PERFORMANCE MEASURES FOR THE SFBSS MODEL.

Performance Measure	Observed Value	Base Case Model Mean	Diff. (%)	95% CI for Base Case Model Mean
Time in System (minutes)				
Annual Riders	9.70 ^a	9.70	0.00 (0.0)	(9.51, 9.89)
Casual Riders	49.95 ^a	49.19	0.76 (1.5)	(46.77, 51.61)
All Riders	12.91 ^a	12.77	0.14 (1.1)	(12.52, 13.02)
Number of Completed Trips				
Annual Riders	1015.4 ^a	1013.1	2.3 (0.2)	(998.4, 1027.8)
Casual Riders	88.0 ^a	85.6	2.4 (2.7)	(82.4, 88.8)
All Riders	1103.4 ^a	1098.7	4.7 (0.4)	(1083.6, 1113.8)
% of Time w/ Shortage				
Bikes	1.85 ^b	4.73	2.88 (-156)	(4.31, 5.15)
Docks	0.95 ^b	4.27	3.32 (-350)	(3.84, 4.70)
Number of Shortage Events				
Bikes	N/A	109.7	N/A	(100.1, 119.3)
Docks	N/A	124.9	N/A	(113.0, 136.8)
Total	N/A	234.6	N/A	(220.4, 246.8)

^a Derived from the operator's Trip data file. ^b Derived from the operator's Status data file.

VI. EXPERIMENTATION

We used SFBSS to investigate some static rebalancing strategies, namely, adding a small number of docks or bikes (or both) to a few stations. In three experiments, we looked at adding only two dozen docks, adding only one dozen bikes, and adding both two dozen docks and one dozen bikes simultaneously, focusing on the impact on the number of bike and dock shortages experienced by riders. Reducing dock shortages will make the BSS less frustrating for riders because they will spend less time searching for a station to return their bikes. As a consequence, riders will spend less time in the system and the availability of bikes for other potential riders will increase. This, in turn, will generate more rides being taken, along with the associated environmental benefits from more people using bikes rather than carbon-based transportation modes. Similarly, reducing bike

shortages directly increases the ability of the BSS to satisfy more demand, so that more rides are taken, raising BSS revenues modestly and lessening environmental impacts. Each experiment used *OptQuest for Arena*, an optimization routine packaged with *Arena* (Kelton et al., 2010) to solve an appropriate mathematical program. Because *OptQuest* uses heuristic methods, it is not guaranteed to find global optimal solutions, only local optima.

6.1. Adding Two Dozen Docks

Imbalances between supply and demand at various stations lead to some stations experiencing dock shortages. In particular, base case model output indicated that five stations had the largest average number of dock shortages per day and accounted for 84% of all dock shortages: stations 70 (with 45.2), 69 (with 28.8), 65

(with 11.3), 64 (with 10.5), and 61 (with 8.7). We set up an optimization model that sought to minimize the total number of dock shortages by adding a total of 24 docks (in increments of 4) at these five stations while holding all other inputs fixed. More formally, letting D_j (d_j) be the number of docks installed (currently) at station j , we instructed *OptQuest* to solve the following optimization problem.

Minimize Number of Dock Shortages

subject to:

$$99 \leq D_{61} + D_{64} + D_{65} + D_{69} + D_{70} \leq 123$$

$$d_j \leq D_j \leq d_j + 24, \text{ integer, for } j = 61, 64, 65, 69, 70$$

After 50 simulations of 30 replications each, the best solution found by *OptQuest* was to add 16 docks at station 70 (which, because it is located at San Francisco's main train depot, actually has room to accommodate this solution) and eight docks at station 65. Results are shown in the second block of rows of Table 2 (bold values indicates statistically significant differences from the base case at $\alpha = 5\%$). For a system with 665 docks, adding 24 docks to the system represents an increase of only 3.6%, but could cut the number of dock shortages by 41%, while slightly increasing the number of bike shortages by 5%. The total number of shortage events drops by 20%.

TABLE 2. SIMULATION RESULTS FROM SMALL INCREASES IN THE TOTAL NUMBER OF BIKES AND DOCKS.

Scenario	Time in System (min.)			Trips Completed per day			Shortages per day		
	Annual	Casual	All	Annual	Casual	All	Bikes	Docks	Total
Base Case	9.70	49.95	12.91	1015.4	88.0	1013.4	109.7	124.9	234.6
Min. Dock Shortages	9.20	48.58	12.27	1008.8	84.9	1093.7	115.4	73.3	188.7
Δ from Base Case	-5%	-1%	-4%	0%	-1%	0%	5%	-41%	-20%
Min. Bike Shortages	9.97	49.55	13.02	1037.5	86.8	1124.3	82.1	148.9	231.0
Δ from Base Case	3%	1%	2%	2%	1%	2%	-25%	19%	-2%
Min. Total Shortages	9.22	49.81	12.34	1027.5	86.1	1113.6	94.2	69.4	163.6
Δ from Base Case	-5%	1%	-3%	1%	1%	1%	-14%	-44%	-30%

6.2. Adding a Dozen Bikes

Supply and demand imbalances also cause bike shortages at various times at some stations. In the model's base case, six stations had the highest average number of bike shortages per day and accounted for 60% of all bike shortages: station 70 (with 23.9), 73 (with 11.1), 55 (with 9.3), 62 (with 8.0), 56 (with 7.9) and 69 (with 6.1). We ran a second optimization to minimize the total number of bike shortages by adding 12 bikes in total (at the start of the day) at these six stations, keeping all other inputs at their base case values. In particular, letting B_j (b_j) represent the number of bikes available (*currently*) at the beginning of each day at station j , we set up *OptQuest* to solve the following problem.

Minimize Number of Bike Shortages

subject to:

$$64 \leq B_{55} + B_{56} + B_{62} + B_{69} + B_{70} + B_{73} \leq 76$$

$$b_j \leq B_j \leq \min(b_j + 12, d_j), \text{ integer, for } j = 55, 56, 62, 69, 70, 73$$

After 50 simulations of 30 replications each, the best solution found was to add six bikes at both stations 55 and 70, yielding performance measures shown in the third block of rows of Table 2. While a dozen bikes in a system of 323 bikes represents only a 3.7% increase, the resulting bike allocation could cut the number of bike shortages by 25%. However, this solution also raises the average number of dock shortages by 19% so, overall, this does not seem to be a solution worth implementing.

6.3. Adding Two Dozen Docks and a Dozen Bikes

Since bikes are best added to the system when docks are added at the same time,

we ran a third optimization to minimize the total number of bike and dock shortages by adding a total of 24 docks and 12 bikes at the aforementioned stations. Specifically, we ran the following model with *OptQuest*.

Minimize Number of Dock + Bike Shortages

subject to:

$$99 \leq D_{61} + D_{64} + D_{65} + D_{69} + D_{70} \leq 123$$

$$64 \leq B_{55} + B_{56} + B_{62} + B_{69} + B_{70} + B_{73} \leq 76$$

$$d_j \leq D_j \leq d_j + 24, \text{ integer, for } j = 61, 64, 65, 69, 70$$

$$b_j \leq B_j \leq \min(b_j + 12, d_j), \text{ integer, for } j = 55, 56, 62, 69, 70, 73$$

After 150 simulations of 30 replications each, the best solution found was to add 20 and 4 docks at stations 70 and 69, respectively, along with 9 bikes at station 55, 2 at station 56, and 1 at station 70. The resulting performance measures (bottom section of Table 2) show that by judiciously adding just 3-4% to the number of docks and bikes in the system, a statistically significant and practically important 30% decrease can be achieved in the total number of dock and bike shortages.

VII. CONCLUSION AND FUTURE WORK

Like most cities, San Francisco's BSS faces a great deal of fluctuation in the demand for bikes and docks at its stations at different times of the day. This affects the imbalance problem by *shifting* the location of the bike and dock shortages throughout the day. For example, SF's busiest station (70) experiences a large number of bike shortages early each weekday morning as numerous commuters

arrive on trains and seek bikes to ride to their final destinations within the city; near the end of each day, this station often runs out of docks as commuters return their bikes before hopping on trains heading out of the city. San Francisco's BSS operator operates trucks that move bikes around the city but they cannot use the same rebalancing strategy all day, e.g., they must deliver many bikes to station 70 in the morning but remove many in the afternoon. This paper has described an animated discrete-event simulation model of a BSS that can be used to investigate key issues faced by BSS operators. In particular, we investigated how changing the daily initial conditions of the system by slightly increasing the number bikes and docks at just a few stations might improve overall system performance and lessen the need for rebalancing. Results based on data from San Francisco's BSS showed that selectively adding 3-4% to the total number of docks and bikes could decrease the total number of dock and bike shortages by 30%. Doing so could reduce the frequency and quantity of truck-based rebalancing activity performed by the operator and its associated environment impact, at relatively low cost.

One possible area for future work would be to study the impact of paying some riders to rebalance bikes by "riding against the tide" to stations that tend to experience bike shortages. Modeling could be used to simulate various pricing strategies that might induce at least some riders to participate in dynamic bike rebalancing. It might also be worthwhile to more thoroughly analyze the Status data file to see if patterns in the BSS operator's rebalancing efforts could be detected and understood; if so, operator-based dynamic rebalancing activity could be incorporated into the simulation model. Finally, as the San Francisco Bay Area gears up for a 10-fold increase in the number of bikes in its system (Green, 2015), we could expand our model and test how these ideas and others might perform in a much larger bike sharing system.

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