Availability Design of Supply Chain Distribution System

Abdel-Aziz M. Mohamed*
Arab Academy for Science, Technology and Maritime Transport, Cairo, Egypt

Distribution system is an important supply chain component which significantly impacts profitability and speed of delivery. This paper presents a simplified approach to help practitioners design a supply chain distribution system or evaluate the availability of an existing one. The research develops a multiple regression model which connects the supply chain distribution design parameters with the overall system availability. The approach used the demand forecast to identify the minimum number of vehicles that must be in operating conditions daily for the system to perform its anticipated work. Three alternatives courses of action were identified, the cost associated with each alternative was estimated, and decision tree was employed to identify the least cost alternative. Regression analysis results reveal that the factors impacting the steady state system availability are: selecting drivers with excellent driving history, increasing the $\frac{n}{k}$ vehicle ratio, increasing the rate of body repair, and the rate of mechanical/electrical repair, respectively.
Keywords: Supply chain, Design of distribution system, Availability, Reliability, Regression Model

* Corresponding Author. E-mail address: amohamed55@yahoo.com

I. INTRODUCTION

Physical distribution refers to all activities responsible of moving products throughout the supply chain flow. Freight transport moves products between the stages of the supply chain from the suppliers to the end customers thereby; impacting both the supply chain efficiency and responsiveness. Enhancing its performance would reap immense increases in the supply chain profitability as well as the speed of delivery.

This research paper considers a transportation system consisting of n vehicles and m drivers. The system needs k functioning vehicles and k drivers to satisfy its daily demand for service delivery. Vehicles fail with exponential failure rates; the repair facility restores the vehicles to their operating state with exponential repair rates. Each driver has a $1-p$ probability of absence. A cost function was proposed to model vehicle purchasing cost, operating cost, and drivers' wages in terms of the vehicle's failure and repair rates as well as the number of drivers hired.

The objective of this research is to present a simple approach to help practitioners design a supply chain distribution system or evaluate the steady state availability of an existing one. The model converts a complex equation into a simple regression model. The coefficients of the regression model provide practitioners with estimates of the contributions of the design parameters to improve the steady state distribution system availability. The method determines the number of required vehicles that must be available on their operating conditions for the system to be able to perform its daily intended distribution work. The fleet size that can provide the required service level will be estimated for different vehicle brands and speed of repair. Decision tree is used to select the most preferred course of action.

II. LITERATURE REVIEW

Availability and reliability are primitive considerations when planning, designing, and operating transportation systems. According to Kapur and Lamberson (1977), reliability is the probability that the system, when operating under stated environmental conditions, will perform its intended function adequately for a specified period of time. They defined availability as the ratio of the operating time of the system to the operating time plus the down time, excluding the idle time. Availability is employed instead of reliability for systems consisting of repairable components as it measures both reliability and maintainability. For instance, suppose a consumer had to choose between a highly reliable product that is difficult to repair and a product that is slightly less reliable but easier to repair. The consumer is more likely to select the product that is slightly less reliable due to the disadvantage the other has of lengthy repair time and less availability.

Elegbede and Adjallah (2003) developed a mathematical model to optimize both system cost and system availability in repairable parallel-series systems. They proposed the following mathematical expressions for system cost $C_s(\lambda, \mu, k)$ and system availability $A_s(\lambda, \mu, k)$:

$$C_s(\lambda, \mu, k) = \sum_{i=1}^{s} k_i \left(\alpha_i \lambda_i^{p_i} + b_i \mu_i^{q_i}\right)$$

$$A_s(\lambda, \mu, k) = \Pi_{i=1}^{s}\left(1 - \left(\frac{\lambda_i}{\lambda_i + \mu_i}\right)^{k_i}\right)$$

Where the failure and repair rates of subsystem i are λ_i and μ_i; the coefficients of the cost function for subsystem i are α_i, b_i, p_i, the number of redundant components in subsystem i is k_i, and the number of subsystems is s. They converted the bi-criteria optimization model into a problem with a single objective function by assigning weights to the functions. The model constraints are on system cost, system availability as well as on upper/lower failure and repair rates. Juang et al. (2008) developed a knowledge-based decision support system to optimize the system cost and availability that determines the components' mean-time between failures (MTBF), mean-time to repair (MTTR) for a series parallel system. They have adapted the following cost function presented in Tillman et al. (1980):

$$C(MTBF) = \alpha\,(MTBF)^{\beta} + \gamma$$

Where $\alpha, \beta, \gamma > 0$, are the parameters of the cost function, and $C(MTBF)$ is the system cost. This cost function was modified to:

$$TC = \sum_{i=1}^{k}\left(\alpha_i\,(MTBF_i)^{\beta_i} + \gamma_i\right) + \sum_{i=1}^{k}\left(a_i - b_i\; MTTR_i\right)$$

Their objective function maximizes the ratio of system availability and system cost:

$$Max. \frac{System\;\; Availability}{TC}$$

Their constraints were upper/lower bounds on the mean-times between failures and mean-time to repair. Genetic algorithm was employed to solve their proposed mathematical model.

Liu (2012) developed a mathematical model to minimize the overall system cost subject to constraints on the upper/lower limits in respective to the steady state availability of each subsystem; a lower limit of the steady state availability of the entire system, upper limits on the system weight and volume. The Tabu search was used to determine the values of the discrete decision variables while the Genetic algorithm was used to determine the values of the continuous variables.

Mohamed (2014) developed an interactive heuristic algorithm to optimize the assembly of coherent systems under two criteria: cost and reliability. The model assumed that the components weren't repairable and that component reliabilities were independent of the location to which the component was assigned.

Feizollahi et al. (2015) developed a robust cold standby redundancy allocation model for series-parallel system. They proposed constraints on budget uncertainty. Liu and Ke (2015) studied the inferences of an availability system. The system consisted of two operating units and one warm standby and has reboot delay and standby switching failures. They assumed that the failure and repair times follow an exponential and a general distribution. They constructed a normal estimator of the availability for the repairable system.

Zoulfaghari et al. (2015) considered a parallel series system. Some subsystems consisted of repairable components and the others consisted of unrepairable components. The system availability is:

$$A_{sys}(t) = \prod_{i\epsilon R}(1-(1-R_t(t))^{n_i}) \prod_{i\epsilon A}(1-(1-A_t(t))^{n_i})$$

Where R is the set of non-repairable components and A is the set of repairable components. They considered the following cost function:

$$cost = \sum_{i=1}^{s} c_i \, n_i$$

Constraints on subsystem weights and costs as well as upper/lower bounds on the number of components permitted on each subsystem. They used the genetic algorithm to optimize the mathematical model.

III. OPTIMIZATION MODEL DEVELOPMENT

This research paper considers a distribution system consisting of a fleet of n vehicles and m drivers in a supply chain. The system could perform its intended functions when k vehicles are available daily in their operating conditions and k drivers are available for work. The system is modeled as a series system consisting of two subsystems: (a) driver subsystem and (b) vehicle subsystem as shown in Figure 1.

3.1. Vehicle Subsystem

Consider a fleet of n vehicles that only $k, k \leq n$ vehicles were enough to perform the

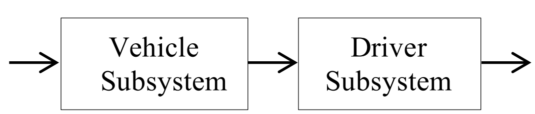

FIGURE 1. BLOCK DIAGRAM.

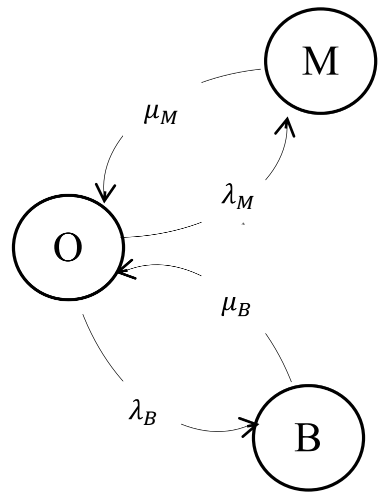

FIGURE 2. TRANSITION DIAGRAM OF A SINGLE COMPONENT SYSTEM.

daily merchandise distribution activities if in operating conditions. Section 3.1.1 presents the derivation of the steady state system availability for a system with a single vehicle and Section 3.1.2 generalizes the result to be used for a multiple vehicles subsystem.

3.1.1. Single Vehicle Subsystem

Consider a system that consists of a single vehicle with a possibility of failure due to mechanical/electrical complications or a possibility of accidents. Vehicle failure distribution is exponential with mechanical/ electrical failure rate λ_M, and vehicle body fails due to accidents with failure rate λ_B. Repair facilities could restore the vehicle to its operating state with exponential repair rate μ_M if the vehicle has a mechanical or electrical problem. The body workshop could restore the vehicle with a repair rate μ_B. The transitional diagram of the single component system is shown in Figure 2. State O represents the state when the vehicle is functioning, state M is the state that the vehicle has mechanical/electrical failure and state B is the state the vehicle has body failure due to accident.

The steady state probability that the vehicle is in the state of mechanical repair is:

$$P_M = \left(\frac{\lambda_M}{\mu_M}\right) P_{O,}$$

The steady state probability that the vehicle is in the state of body repair is:

$$P_A = \left(\frac{\lambda_B}{\mu_B}\right) P_O.$$

Since the sum of probabilities of a vehicle to be in one of the three states is 1;

$$\left(\frac{\lambda_M}{\mu_M}\right) P_O + \left(\frac{\lambda_B}{\mu_B}\right) P_O + P_O = 1$$

The steady state availability of one single vehicle is:

Availability = $P_O = \frac{\mu_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}$

3.1.2. Multi-vehicle Subsystem

This subsection considers a multi-vehicles subsystem. It assumes that it requires k functioning vehicles for the subsystem to be able to perform its intended function appropriately. If the number of functioning vehicles is less than k, the subsystem will fail to perform its intended function and the entire subsystem will be in the state of failure. When one or more vehicles are repaired, the subsystem state will be considered functioning.

3.1.2.1. Steady State Vehicle Subsystem Availability

This subsection presents the steady state availability model for the vehicle subsystem. The subsystem consists of n vehicles. It can perform its daily work if only k vehicles are in the operating conditions. Vehicle failure rates are assumed to be independent and identically distributed random variables as well as the repair rates of the repair facility. The availability of the vehicle subsystem is modeled as a k-out-of-n system. The steady state subsystem availability function is illustrated as:

$$A(\lambda,\mu)=\sum_{i=k}^{n}\binom{n}{i}\left(\frac{\mu_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}\right)^i\left(\frac{\lambda_M\mu_B+\lambda_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}\right)^{n-i}$$

3.1.2.2. Expected Number of Operating Vehicles, Failed Vehicles, and Their Standard Deviations

The expected numbers of operating vehicles are to be determined as follows:

$$\mu_o=n\frac{\mu_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}$$

Where μ_o is the expected number of operating vehicles in a specific period of time T and the expected number of failed vehicles in the same period of time is $n-\mu_o$. The variance of the number of operating vehicles can be determined as:

$$\sigma^2=n\left(\frac{\mu_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}\right)\left(\frac{\lambda_M\mu_B+\lambda_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}\right)$$

Example 1: A transportation system of 100 vehicles. Vehicle mechanical failure rate is .03 failures per day and 0.002 failures due to accident per day. The repair rates are 3 mechanical repairs and 0.2 vehicle body repairs daily. What is the expected number of functioning vehicles daily and their variance daily?

$$\mu_o=n\frac{\mu_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}=100\frac{0.2*3}{0.03*0.2+0.002*3+0.2*3}=98$$

Variance = 100 *0.98 *(1 - 0.98) =1.96

Standard deviation = $\sqrt{1.96}=1.4$

3.1.2.3. Steady State System Availability for the case when $k=n$

In this case, the minimum number of vehicles k is assumed to be equal to the vehicles fleet size n, the steady state system availability will be:

$$A(\lambda,\mu,p)=\sum_{i=n}^{n}\binom{n}{i}\left(\frac{\mu_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}\right)^i\left(\frac{\lambda_M\mu_B+\lambda_B\mu_M}{\lambda_M\mu_B+\lambda_B\mu_M+\mu_B\mu_M}\right)^{n-i}$$

$$A(\lambda,\mu,p) = \sum_{i=n}^{n} \frac{n!}{n!\,(n-n)!} \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^i \left(\frac{\lambda_M\mu_B + \lambda_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{n-i}$$

$$A(\lambda,\mu,p) = \sum_{=n}^{n} 1 \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^n \left(\frac{\lambda_M\mu_B + \lambda_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{n-n}$$

The $k-out-of-n$ model is reduced to the following:

$$A(\lambda,\mu,p) = \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^n$$

We can conclude that when $(k = n)$ the system is reduced to a series system.

3.1.2.4. Steady State System Availability for the Case when $k = 1$

The second special case is to assume that $k = 1$,the transportation system availability is

$$A(\lambda,\mu,p) = \sum_{i=1}^{n} \frac{n!}{i!(n-i)!} \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^i \left(\frac{\lambda_M\mu_B + \lambda_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{n-i}$$

$$= \left(1 - \frac{n!}{0!\,(n-0)!} \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^i \left(\frac{\lambda_M\mu_B + \lambda_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{n-i}\right)$$

$$= 1 - \left(1 - \frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{n-i}$$

This concludes that when $(k = 1)$ the system is reduced to a parallel system.

3.2. Driver Subsystem

The reliability for the m drivers that could adequately perform the job when k are present is modeled as a k-out-of-m sub-system of drivers. The probability that a driver is available for work in any given day is p. The probability that an employee is absent is q that is estimated from the HR absent record for vehicle drivers where $p = 1 - q$. Drivers The probability that a driver will attend his/her work day is assumed to be independent because his/her absence will not influence other drivers to do the same. The drivers' subsystem reliability is:

$$R(p) = \sum_{i=k}^{n} \binom{m}{i} p^i\,(1-p)^{m-i}$$

3.3. Numerical Example

This example illustrates the utilization of the developed model to compute the steady state system availability for a transportation system consisting of 15 vehicles and 18 drivers. The system could perform its intended assignment in a specific day if 13 trucks are in their operating conditions and 13 drivers are available for work. Truck failure rate is 0.03 failures per day and the accident rate is 0.002 per day. The probability that a driver comes to work is 0.96. Mechanical repair rate is 3 per day and the body shop is 0.1 per day. The steady state availability of a vehicle is determined as:

$$P_O = \frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}$$

$$\frac{0.1 * 3}{0.03 * 0.1 + 0.002 * 3 + 0.1 * 3}$$

$$= \frac{0.3}{0.003 + 0.006 + 0.3} = 0.970873786$$

The steady state availability of the vehicle sub-system is:

$$A(\lambda,\mu) = \sum_{i=13}^{15} \binom{15}{i} \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^i \left(\frac{\lambda_M\mu_B + \lambda_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{15-i}$$

$$A(\lambda,\mu) = \sum_{i=13}^{15} \binom{15}{i} (0.970873786)^i (1 - 0.970873786)^{15-i}$$

$$A(\lambda,\mu) = Prob(x = 13) + Prob(x = 14) + prob(x = 15)$$
$$A(\lambda,\mu) = 0.060656 + 0.288838 + 0.641862 = 0.991356$$

Driver sub-system reliability is:

$$R(p) = R(p) = \sum_{i=k}^{n} \binom{m}{i} p^i (1-p)^{m-i} = \sum_{i=13}^{18} \binom{18}{i} p^i (1-0.96)^{18-i}$$

The reliability of the driver subsystem = Prob. (X =13) + Prob. (X =14) + … +Prob. (X=18) =0.99995
The distribution system availability is: 0.99995 * 0.991356 = 0.991306

3.4. Optimization Model

The steady state availability of the system shown in the block diagram in Figure 1 is to be presented. Since the system consists of two independent series subsystems, the steady state system availability is:

$$A(\lambda,\mu,p) = \sum_{i=k}^{n} \binom{m}{i} p^i (1-p)^{m-i}$$
$$\sum_{i=k}^{n} \binom{n}{i} \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^i \left(\frac{\lambda_M\mu_B + \lambda_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{n-i}$$

This function determines the probability that there are k vehicles and k drivers are available for work at the same time. In this model, k is a constant that is identified according to the distribution system workload. The decision variables of the model are: $n, m, \lambda_m, \lambda_b, \mu_m, \mu_b$.

3.4.1. System Cost Constraint

This cost function determines the purchasing cost of the fleet of vehicles, the vehicle operating cost, and driver's salary.

$$TC = \sum_{i=1}^{k} \left(\alpha_i \,(MTBF_i)^{\beta_i} + \gamma_i\right) + \sum_{i=1}^{k} (a_i - b_i\, MTTR_i) + \sum_{i=1}^{m} c_i\, p^{\alpha} \le uc$$

The variable uc is an upper limit on the total system cost. The operating costs are the cost of operating, maintaining and repairing the fleet of vehicles. The fleet operating costs depend on the vehicles' failure and repair rates. The driver salary depends on the experience the drivers have.

3.4.2. System Availability Constraints

This subsection presents the mathematical expression used to determine the reliability of the driver subsystem. Constraints on driver reliability such as:

$$\sum_{i=k}^{n} \binom{m}{i} p^i (1-p)^{m-i} \ge p_l$$

And constraints on vehicle subsystem availability:

$$\sum_{i=k}^{n} \binom{n}{i} \left(\frac{\mu_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^i \left(\frac{\lambda_M\mu_B + \lambda_B\mu_M}{\lambda_M\mu_B + \lambda_B\mu_M + \mu_B\mu_M}\right)^{n-i} \ge A_l$$

Where p_l and A_l are lower limits assigned to the driver subsystem reliability and vehicle subsystem availability respectively. Upper and lower bounds on each failure and repair rate:

$\lambda_i^l \le \lambda_i \le \lambda_i^u$, $\mu_i^l \le \mu_i \le \mu_i^u$ for all $i = 1, 2, \dots n$

IV. REGRESSION ANALYSIS MODELING

This section presents a multiple linear regression model to predict the steady state system availability. The regression model is meant to replace the complicated steady state system availability equation presented in Section 3.4 to help practitioner use the quantitative tools when designing the supply chain distribution system. The regression model coefficients show the estimated values of the contribution of each design parameter on the overall steady state system availability. The data used in the regression model are computed from the optimization model presented earlier at different randomly generated values of the variables: n, k, λ_{M}, λ_{B}, μ_{B}, *and* μ_{M}. The sample size used in the regression model is 1116 lines of data. The minimum number of vehicles k is selected to be a value between 1 to 30 vehicles. For each value of k, several values of the number of vehicles n are selected $k \leq n \leq k+10$. Mechanical failure rates, auto-body failure rates (accident rate), and the rates of repair of the respective repair facilities are randomly generated from uniform probability distribution functions between the identified limits presented in Table 1.

The model dependent variable Y is the steady state system availability. The independent variables are the number of vehicles n, the minimum number of vehicles required for the system to perform its intended function k, mechanical and body failure rates λ_M, λ_B and mechanical and body repair rates are μ_M, μ_B respectively. The multiple linear regression model representing the population can be stated as follows:

$$Y_{ij} = \beta_0 + \beta_1 n_i + \beta_2 k_i + \beta_3 \lambda_{Mi} + \beta_4 \lambda_{Bi} + \beta_5 \mu_{Mi} + \beta_6 \mu_{Bi} + \epsilon_{ij}$$

β_i is the coefficient of the i^{th} independent variable for $i = 1, 2, \dots, 6$; ϵ_{ij} is the error term which is assumed to be normally distributed random variable with mean equal 0. The regression equation that can be estimated from sample data set is:

$$\bar{y} = b_0 + b_1 n_i + b_2 k_i + b_3 \lambda_{Mi} + b_4 \lambda_{Bi} + b_5 \mu_{Mi} + b_6 \mu_{Bi}$$

Where b_i is the estimate of β_i for $i = 1, 2, \dots, 6$. Using the generated data, the estimated regression equation is:

$$\bar{y} = 0.7158 + 0.124 n_i - 0.158 k_i - 1.551 \lambda_{Mi} - 7.51 \lambda_{Bi} + 0.132 \mu_{Mi} - 0.109 \mu_{Bi}$$

TABLE 1. UNIFORM DISTRIBUTION PARAMETERS USED TO GENERATE DATA THE RGRESSION MODEL.

Parameter	Lower Limit	Upper limit
λ_M	0.01	0.08
λ_B	0.01	0.08
μ_M	0.2	5.0
μ_B	0.06	0.7

TABLE 2. DATA ANALYSIS REGRESSION OUTPUT.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.945425
R Square	0.893828
Adjusted R Square	0.893253
Standard Error	0.106904
Observations	1116

ANOVA

	Df	*SS*	*MS*	*F*	*Significance F*
Regression	6	106.6989	17.78315	1556.046	0
Residual	1109	12.67412	0.011428		
Total	1115	119.373			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.715833	0.02123	33.71781	3.9E-172	0.674177	0.757489
K	-0.15845	0.00178	-88.9988	0	-0.16194	-0.15495
N	0.124198	0.001495	83.09075	0	0.121266	0.127131
lambda-m	-1.55162	0.217636	-7.12944	1.81E-12	-1.97864	-1.1246
mu – M	0.132604	0.021266	6.235617	6.39E-10	0.090879	0.174329
lambda – B	-7.51892	5.910579	-1.27211	0.2036	-19.1161	4.078259
mu –b	-0.10932	0.978479	-0.11172	0.911063	-2.0292	1.81056

4.1. Model Validity

In this phase, the validity of the regression model is to be verified. The linearity assumptions of the model and the residual probability distribution were tested.

4.1.1. Distribution of Residuals

The error term used in the regression model is assumed to be normally distributed random variable with mean 0 and standard deviation σ. The normal probability plot of residuals shown in Figure 3 is nearly linear which suggests that the residuals are normally distributed. The following hypothesis is to test if the mean of the error is zero.

$$H_0: \mu = 0, H_1: \mu \neq 0$$

Statistical measures of residual data show the following:

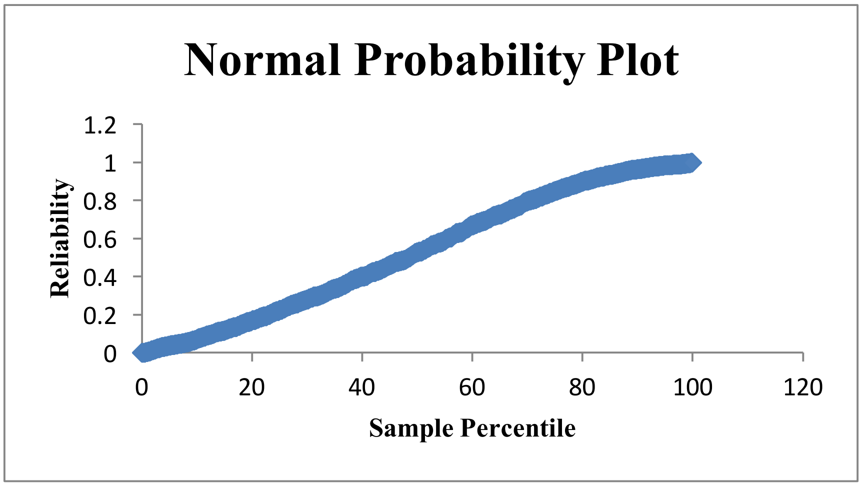

FIGURE 3. NORMAL PROBABILITY PLOT.

Average =-1.17857E-15
Standard deviation = 0.106615
Number of observations = 1116

The result of the calculation of the t value is $t \cong 0$. These results suggest not to reject the null hypothesis. Therefore $\mu = 0$.

4.1.2. Multicollinearity

Multicollinearity exists in the multiple linear regression analysis when the independent variables are highly correlated. It creates several problems in the estimated regression model. One of the major problems is that it concludes several possible sets of values of the regression coefficients may possibly fit the regression equation. As a result, the interpretation of the estimated values of the regression coefficients deem invalid since several values exist for each coefficient.

The Variance Inflation Factor (VIF) test, (Lind et al., 2008, pages 533-535) is used to examine the correlation between the independent variables. If the VIF test value is greater than 10, the independent variables are highly correlated. The test is conducted as follows:

$$VIF = \frac{1}{1 - R_j^2} = \frac{1}{1 - .93} = 14.3$$

The term R_j^2 is the maximum coefficient of determination when an independent variable is used as a dependent variable and the remaining independent variable remain independents. The VIF result indicates that the independent variables are highly correlated and as a result multicollinearity exists in the model.

4.1.3. Remedy of the multicollinearity

The VIF test reveals that there is a strong link between the independent variables. That correlation illustrates that the independent variables n and k are highly correlated. The transformation approach was utilized to avoid the correlated variables. The variables k and n with a coefficient of correlation 0.9634 were replaced by the variables n and $\frac{n}{k}$ that have a coefficient of correlation of 0.0225. The table below shows the new data analysis regression output. Since VIF has lesser values than 10, the independent variables in the current model don't correlate

and consequently can be used in the regression model.
Using the new regression data, the regression equation, Table 3, is:

$$\hat{y} = -1.106 - 0.00425n_i + 1.3386\frac{n_i}{k_i} - 0.6446\lambda_{Mi} - 5.139\,\lambda_{Bi} + 0.05\mu_{Mi} + 0.86\mu_{Bi}$$

TABLE 3. NEW DATA ANALYSIS REGRESSION OUTPUT.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.867896
R Square	0.753244
Adjusted R Square	0.751909
Standard Error	0.170951
Observations	1116

ANOVA

	Df	*SS*	*MS*	*F*	*Significance F*
Regression	6	98.93312	16.48885	564.2193	0
Residual	1109	32.40963	0.029224		
Total	1115	131.3428			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-1.1063	0.039623	-27.9205	2.4E-130	-1.18404	-1.02855
lambda-m	-0.64466	0.322941	-1.99622	0.046155	-1.2783	-0.01102
mu – M	0.050465	0.005715	8.829685	4.02E-18	0.039251	0.061679
lambda – B	-5.13912	0.292264	-17.5838	3.07E-61	-5.71257	-4.56567
mu –b	0.861068	0.035622	24.17255	4.9E-104	0.791174	0.930961
N	-0.00425	0.000683	-6.21731	7.15E-10	-0.00559	-0.00291
n/k	1.33861	0.027445	48.77475	3.3E-278	1.284761	1.392459

4.1.4. Case 1: when $n = k$

When $n = k$, i.e no back-up vehicles exist in the system, the ratio $\frac{n}{k} = 1$, the system availability will be portrayed as:

$$\hat{y} = 0.232 - 0.00425n_i - 0.6446\lambda_{Mi} - 5.139\lambda_{Bi} + 0.05\mu_{Mi} + 0.86\mu_{Bi}$$

If the size of the fleet n is increased by one vehicle while the ratio remains 1, the system availability will be reduced by 0.00425. When the rate of mechanical repair for a vehicle steadily increases by one per day, the system availability will increase by 0.05, and the body repair rate will increase the system availability by 0.86 for each unit increases over in the existing body repair rate.

Using vehicles with lower rate of failure and hiring professional drivers with lower rate of accidents will improve system availability more than all other factors.

4.1.5. Case 2: when n > k

When n > k, there is $n-k$ redundant vehicles exist in the system, the system availability is expressed as:

$$\hat{y} = -1.106 - 0.00425n_i + 1.338\frac{n_i}{k_i} -0.6446\lambda_{Mi} - 5.139\,\lambda_{Bi} + 0.05\mu_{Mi} +0.86\mu_{Bi}$$

If the fleet ratio $\frac{n_i}{k_i}$ increases, two cases may occur. The first case is that one in which the fleet ratio increases due to a decrease in the k value, the fleet availability will go up by 0.1338 for each 0.1 unit increase in the ratio. If the ratio increase was due to the increase in the number of vehicles n; the system availability will increase by 0.1338 and decrease by 0.00425 times the increase in the number of vehicles. When the rate of mechanical repair for a vehicle goes up by 0.1 per day, system availability will increase by 0.005, and the body repair rate will increase the system availability by 0.086 for each 0.1 unit increases in the body repair rate. Using vehicles with lower rate of failure and employing drivers with lower rate of accidents will improve system availability the most.

V. DESIGN OF THE SUPPLY CHAIN DISTRIBUTION SYSTEM

This section presents a simplified approach to design a new supply chain distribution system or to evaluate the steady state availability of an existing one. The approach identifies the design parameters such as the number of vehicles required (fleet size) n, number of drivers, vehicle failure rates, rates of both the mechanical and body repair facilities that maintain a specific system availability. Section 5.1 determines the minimum number of vehicles k that are needed to be operating every day for the distribution system to perform their required functions.

5.1. Number of Vehicles Required Performing the Anticipated Work-Load

A reliable forecasting technique is employed to predict the future distribution demand of the supply chain. Historical data of the supply chain distribution loads in terms of ton-kms, cubic-meter-kms, pound-miles, or cubic-yard-miles are to be prepared for demand forecast. If the weight criterion is considered, the weight of each order is to be multiplied time the delivery distance. All weight delivery distances or volume delivery distances are aggregated for deliveries as needed – weekly, monthly, or quarterly as seen in Table 4 below.

Data of any irregular variations originated due to the occurrence of irregular events are to be disposed of. Seasonality should, then, be assessed. If it exists in the data; information in respective to that seasonal timing are to be excerpt and collected to further determine the seasonal relatives. The aftermath includes employing the appropriate method of forecasting to predict the future distribution load without seasonality.

Seasonality is, then, added to the predicted demand of the forecast F_t at time t. Given the standard vehicle weight-distance capacity, WDC, and the standard vehicle volume-distance capacity, VDC, the minimum number of vehicles k required to perform the distribution work can be determined as followed:

$$k = Max\left\{\frac{\sum_{i=1}^{n} w_i l_i}{WDC}, \frac{\sum_{i=1}^{n} v_i l_i}{VDC}\right\}$$

The result of the above equation is to be rounded up to the nearest integer value.

TABLE 4. AGGREGATE HISTORICAL DISTRIBUTION DEMAND.

Period	Weight	Volume	Distance to customers	Wight. Distance	Volume. distance
1	w_1	v_1	l_1	$w_1 l_1$	$v_1 l_1$
2	w_2	v_2	l_2	$w_2 l_2$	$v_2 l_2$
3	w_3	v_3	l_3	$w_3 l_3$	$v_3 l_3$
n	w_n	v_n	l_n	$w_n l_n$	$v_n l_n$
			Aggregate historical demand	$\sum_{i=1}^{n} w_i l_i$	$\sum_{i=1}^{n} v_i l_i$

5.2. System Design When the Vehicle Failure and Repair Rates are Discrete

In the real life applications, most of the decision variables in the supply chain distribution system are discrete. Vehicle failure rates in practice are not continuous variables, they depend on the vehicle brands available in the market if buying new vehicles; or the vehicle brands and conditions if buying used vehicles. In such a case, a set of decision alternatives can be generated and the Decision tree can be employed to determine the best course of action. An alternative approach is to develop an integer programming model with binary variables.

5.2.1. Decision Tree Approach

In this case, a study of the available vehicle brands in the market will be carried out to identify the failure rates, the purchasing costs and the expected annual operating costs for each vehicle brand of the ones that may be considered for the fleet. The vehicle failure rates will be categorized into three categories, vehicles with low failure rates, vehicles with medium failure rates, and vehicles with high failure rates. A vehicle is selected from each category based on a set of criteria identified by the users; including the cost-availability ratio. The Factor Rating Method, Stevenson (2012, Page 353), can be employed to identify the optimal vehicle selection from each category. Let us assume the vehicle selected from the set of vehicles in the i^{th} category has mechanical/electrical failure rate $\lambda_{Mi,}$ purchasing cost PC_i and expected annual cost of operation OC_i, where $i = 1, 2, 3$.

The results of the regression model reveal that the rate of body failure which depends on the driver accident rates has the strongest impact on the system steady state availability. Therefore, it is recommended to hire drivers with excellent safe driving record. The failure rate of the vehicle body can be estimated as the average driver rate of being involved in car accidents. Therefore, the decision variables are mainly the automobile purchasing decision and the repair facility rate/size decisions.

Those decisions are considered in the formulation of the following set of alternative courses of action:

1. Purchase vehicles from category 1 and establish a low speed mechanical repair facility.
2. Purchase vehicles from category 2 and stablish an average speed mechanical repair facility.
3. Purchase vehicles from category 3 and establish a fast mechanical repair facility.

5.2.2 Fleet Size Determination

This section computes the fleet size associated with the required steady state system availability when the vehicle purchasing decision is to select alternative $i, i = 1, 2, 3$. In Section 5.1 the value of k was identified based on the anticipated demand. If the management requires the distribution system to have the system availability of, $avail$.

$$-1.106 - 0.00425 n_i + 1.338 \frac{n_i}{k} - 0.6446 \lambda_{Mi} - 5.139 \lambda_{Bi} + 0.05 \mu_{Mi} + 0.86 \mu_{Bi} \geq avail$$

All independent variables are identified in the previous steps except n_i which can be determined for each category based on the selected vehicle failure rate in each category as well as the repair rates. The fleet size recommended if the vehicles purchased belongs to category $i, i = 1, 2, 3$ is:

$$n_i = \frac{avail + 1.106 + 0.6446 \lambda_{Mi} + 5.139 \lambda_{Bi} - 0.05 \mu_{Mi} - 0.86 \mu_{Bi}}{\frac{1.338}{k} - .00425}$$

And the result is to be rounded up to the nearest integer value.

5.2.3. Costs Associated with each Decision Alternative

This section develops the necessary formula to determine the total costs associated with each decision alternative. The purchasing cost of the vehicles will be divided by the expected vehicle lifetime in years lv to determine the annual fixed cost $\frac{n_i PC_i}{lv}$ for all vehicles purchased, where $n_i PC_i$ are the total purchasing costs of all vehicles if they belong to category i. The expected annual operating costs of the vehicles belonging to category i is $n_i OC_i$. Therefore, the total annual costs of purchasing and operating a fleet of vehicles from those that belong to category i is $\frac{n_i PC_i}{lv} + n_i OC_i$

The costs of establishing fast, medium or slow repair facility are assumed as $RF, RM, and\ RS$ respectively. If we assume the lifetime of the repair facilities is constant, lr, the annual fixed cost of the fast, medium and slow repair facilities are $\frac{RF}{lr}, \frac{RM}{lr}, \frac{RS}{lr}$ respectively. The expected annual operating costs for the fast, medium or slow repair facilities are ROF, ROM, ROS. The annual costs of establishing and operating a fast repair facility is $\frac{RF}{lr} + ROF$. The annual costs of establishing and operating a medium repair facility is $\frac{RM}{lr} + ROM$. The annual costs of establishing and operating a slow repair facility is $\frac{RS}{lr} + ROS$.

The cost associated with the courses of action is as follows:

1. Purchase vehicles from category 1 (low failure rates) and establish a low speed mechanical repair facility:
   $\frac{n_1 PC_1}{lv} + n_1 OC_1 + \frac{RS}{lr} + ROS$
2. Purchase vehicles from category 2 (medium failure rates) and stablish an

average speed mechanical repair facility:
$\frac{n_2 PC_2}{lv} + n_2 OC_2 + \frac{RM}{lr} + ROM$

3. Purchase vehicles from category 3 (high failure rates) and establish a fast mechanical repair facility:
$\frac{n_3 PC_3}{lv} + n_3 OC_3 + \frac{RF}{lr} + ROF$

The alternative course of action with the least cost is the one to be selected.

VI. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This paper presents a simplified approach to design a new supply chain distribution system or evaluate the availability of an existing one. The anticipated future supply chain distribution load is to be forecasted. The minimum number of vehicles that are needed to be operable daily for the system to perform their intended distribution functions is identified.

A regression model is introduced as a simplification of the optimization model to connect the supply chain distribution design parameters with the overall system availability; which helps supply practitioner use a simple quantitative tool to design the supply chain distribution system.

When the decision variables are continuous, optimal distribution system design decisions are determined using the proposed optimization model. When the decision variables are discrete, a set of alternative courses of action are evaluated using a decision tree and the least cost alternative is selected.

Regression analysis results reveal that the factors impacting the steady state system availability are selecting drivers with excellent driving history, increasing the $\frac{n}{k}$ vehicle ratio, increasing the rate of body repair, and the rate of mechanical/electrical repair, respectively.

Acknowledgment

The author would like to thank the anonymous reviewers for their constructive comments and suggestions that help improve the quality of the manuscript.

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