An Optimization Approach to Time-Segmented Regression with Application on Capturing the Impact of Supply Chain Disruption on Financial Markets

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Time-Segmented Regression studies multivariate time series data where structural changes in the relationship model occur over time. It has a broad spectrum of application domains. In this research, we study Time-Segmented Regression as a data modeling problem with the objective of developing an efficient optimization approach to piece-wise model building for multivariate time series data. The splitting of data for capturing structural changes is constrained to be done in the time dimension. With an application analyzing systematic risks in financial markets, we demonstrate that our proposed algorithm can efficiently capture the structural changes in systematic risks of stocks caused by supply chain disruptions.

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I. INTRODUCTION

Segmented Regression (SR) is a statistical method used to model the relationship between a dependent variable and one or more independent variables, considering the potential presence of structural changes in the relationship. Time-Segmented Regression (TSR) is a special case of SR primarily used in the context of time series data, where one of the independent variables is time, and the structural changes occur at specific time points. In general SR, on the other hand, the independent variable associated with identifying the breakpoints for structural changes can be any continuous variable, not necessarily time.

In this research, we address the question of how to solve the TSR problem as a data modeling problem in an efficient and effective manner. Two data modeling problems that are related to TSR are Grouped Regression (GR) and Regression Clustering (RC). GR divides the data into groups or categories, often based on a categorical variable, and models the relationship within each category. While the groups/categories can be determined by time, the divides are predetermined. RC combines clustering and regression techniques to group observations and model the relationship within each group. These data analytics methods have valuable applications across a broad spectrum of domains such as finance, economics, social studies, environmental studies, medicine and healthcare, marketing, as well as production and service operations management. The choice among these methods depends on the research question, the nature of the data, and the desired outcome. In this research, we focus on TSR, a special case of SR, that is different from GR and RC problems. The goal is to develop an optimization approach to piece-wise regression model building for multivariate time series data where the splitting of data must be done in the time dimension. The decision variables are the number of segments as well as the breakpoints.

This research on TSR is motivated by the application of time-segmented beta analysis, in which the key statistic is often referred to as "beta" in finance and investment. We are specifically interested in the detection of the financial market reaction to supply chain disruptions from COVID-19.

Beta is an important measure of the systematic risk or market risk of an individual security, such as a stock or a portfolio, relative to the overall market. It is a key component of modern portfolio theory and is widely used to assess the risk-return tradeoff of investments. To model the sensitivity of an individual security's returns to the returns of the overall market or a benchmark index, we use the individual security's returns as the dependent variable and the returns of the market index (e.g., S&P 500) as the independent variable to perform a simple linear regression analysis. The beta is simply the slope of the line. A beta value greater than 1 indicates that the security is more volatile than the market, while a beta value less than 1 suggests that the security is less volatile. A beta of approximately 1 means that the security moves in line with the market. A negative beta means that the security's price tends to increase when the overall market falls, and vice versa. If the beta is not significantly different from zero, the performance of the security or portfolio is not correlated with the overall market. With a zero-beta, the investment has no systematic risk, but it still has inherent idiosyncratic risk. In this research, we focus on the analysis of systematic risk as it is a key component of the Capital Asset Pricing Model (CAPM).

Systematic risk can change over time due to various external and internal reasons including economic cycles, monetary and fiscal policies, regulatory shifts, geopolitical events, technological changes, strategy and business model changes, financial leverage, management changes, risk management practices, as well as operations and supply chain management practices. The COVID-19 pandemic has led to significant supply chain disruptions worldwide, affecting businesses across various industries. These disruptions have, in turn, altered systemic risks in the global economy and capital markets.

Supply chain disruptions have created increased volatility and uncertainty in the global economy. Businesses have faced difficulties in procuring raw materials, components, and finished goods, leading to production slowdowns, inventory shortages, and delays in order fulfillment. This uncertainty has translated into increased systemic risk, as it affects the overall market sentiment and investor confidence.

The pandemic has exposed vulnerabilities in global supply chains, prompting companies and governments to reassess their reliance on specific regions or countries for critical goods and services. This has led to shifts in global trade dynamics, such as the reshoring of manufacturing activities, regionalization of supply chains, and increased focus on supply chain resilience. These changes can alter the systemic risks associated with global trade and economic interdependencies.

Supply chain disruptions have also placed significant financial strain on businesses, especially those with high fixed costs or limited cash reserves. This financial stress can lead to bankruptcies, layoffs, and reduced capital investments, all of which contribute to increased systemic risk in the economy.

In addition, the pandemic has accelerated the adoption of technology across various industries, as businesses seek to improve their supply chain resilience and efficiency. This rapid technological transformation may create new systemic risks, such as increased reliance on digital systems and potential vulnerabilities to cyberattacks.

Supply chain disruptions during the pandemic have led to changes in consumer behavior, too. Online shopping increased, discretionary spending reduced, and product preferences shifted. These changes may create new systemic risks for businesses that are unable to adapt to evolving consumer demands and market trends.

It is crucial for businesses, policymakers, and investors to understand these risks and develop strategies to mitigate their impact and build more resilient supply chains for the future. It is also important for investors to capture the timing of these systematic changes and price them into the valuation of securities. TSR can be a useful analytical tool to help capture the changes of systematic risks as time-segmented beta analysis.

With an application analyzing such dynamics of systematic risks in financial markets, we demonstrate that our proposed TSR model and algorithm can efficiently capture the structural changes in systematic risks of stocks caused by supply chain disruptions.

The rest of this paper is organized as follows. Section II provides a review of relevant literature. Section III defines the TSR data modeling problem as an optimization problem. Section IV proposes a fast heuristic algorithm for solving this optimization problem. Case studies using real financial market data are presented in Section V. Conclusion remarks are drawn in Section VI.

II. LITERATURE REVIEW

The Chow Test is one of the earliest methods used to identify structural breaks in time series data (Chow, 1960). This method tests the null hypothesis that the regression coefficients are constant across different segments. However, it requires prior knowledge of the break points, limiting its applicability in real-world situations (Zeileis, 2005). The Bai-Perron Test is an extension of the Chow Test that allows for multiple breaks in the regression model (Bai and Perron, 1998). This method is particularly useful for detecting multiple structural breaks without prior knowledge of their occurrence. The computational complexity of TSR depends on the number of breakpoints and the functional form within each segment. In general, the estimation of breakpoints can be computationally intensive, especially when dealing with many breakpoints or complex regression functions. However, recent advances in breakpoint detection methods, such as the sequential estimation approach proposed by Qu and Perron (2007), have improved the computational efficiency of these models.

TSR concepts have been applied in various research fields, including economics (Acemoglu et al., 2001; Hartwig, 2012; Kar et al., 2013; Clarida et al., 1999; Mavroeidis, 2010; Cogley and Sargent, 2005; Bergstrand, 1985; Frankel and Romer, 1999; Ravallion and Chen, 1997; Dollar and Kraay, 2002; Hamilton, 1983; Kilian, 2009), social sciences (Narayan and Smyth, 2004; Zeileis et al., 2003), healthcare (Li et al., 2011; El Bcheraoui et al., 2015; Huang et al, 2013; Keogh et al., 2023; Menendez et al., 2015; Stensrud et al., 2019), process and quality control (Montgomery and Mastrangelo, 1991; Chen et al., 2010; Zou et al., 2007; Nelson, 1990; Murthy et al., 2004), Lean and Six Sigma (Womack and Jones, 2010), supply chain management (Lee et al., 1997), and finance (Fama and French, 1993; Lettau and Ludvigson, 2001; Bai and Perron, 2003; Kim and Perron, 2009; Baum et al., 2001; Lothian and Taylor, 2008; Hodrick and Prescott, 1997; Laubach and Williams, 2003; Ang and Bekaert, 2002; Guidolin and Timmermann, 2008).

Sun and Varshney (2019) studied a special case of TSR with univariate time series data. Time-segmented trendlines as well as support and resistance lines were drawn for stock price data and a heuristic algorithm was developed to efficiently determine the number of segments and breakpoints in the multi-objective optimization problem. Oliver et al. (1998) employed TSR to identify behavioral changes in time series data containing a predetermined number of change points. Hébrail et al. (2010) apply dynamic programming algorithms to put functional data into a specified number of piecewise clusters. Chung et al. (2004) and Chen et al. (2013) use genetic algorithms to split time series data, defining the fitness of the solution based on the vertical distances between critical points. Ahmed et al. (2010) suggest that the Classification and Regression Trees (CART) machine learning algorithm can segment time series in a least-squares context. Bryant and Duncan (1994), Duncan and Bryant (1999), Ge and Smyth (2000), and Ge and Smyth (2001) implement dynamic programming approaches to detecting change points in linear regression. Guralnik and Srivastava (1999) utilized an iterative algorithm to decide whether time series segments should undergo further partitioning.

There lacks a study on how to efficiently conduct TSR on multivariate time series data. Given the computational complexity introduced by the additional dimensions, there is a need for developing fast heuristics while pursuing solutions using multiple criteria. Unlike the method in Qu and Perron (2007) that focuses on detecting common breakpoints across multiple regression equations, our goal is to efficiently segment multivariate time series data with an integrated regression approach. This paper's original contributions to the literature are:

1) Proposing a multi-objective nonlinear optimization approach to modeling multivariate time series data with TSR
2) Proposing a fast greedy heuristic to solve the multivariate TSR problem
3) Demonstrate the effectiveness of the method in a unique context by analyzing financial market data to capture the impact of supply chain disruptions on systematic risks

III. PROBLEM DEFINITION

In this paper, the TSR is modelled as a multi-objective optimization problem. The first objective is defined in an ordinary least squares (OLS) sense. That is, we would like to see the minimal sum of squared errors because of the final statistical model. This objective function makes the optimization model nonlinear. The second objective is to minimize the number of segments. This objective is for practical purposes to improve interpretability of the resulting model. The decisions are to determine the number of segments and the split points. Given that one of the features of the multivariate data set is time (e.g., day, week, month), the splitting must be done in the time dimension. Additional constraints of the optimization problem include 1) the number of segments must be greater than or equal to two, and 2) the size of the smallest segment must be greater than or equal to a pre-defined minimal segment size. Table 1 presents the mathematical notation list.

TABLE 1. NOTATION LIST

D	Multivariate Time Series Data Set
N	Total Number of Data Points (Rows) in D
i	Sequential Time Index in D
d_i, $i= 1 \ldots N$	Data Point (Row) i in D
Y	Response Variable Column in D
X	Independent Variable(s) Column(s) in D
m	Number of Segments
k=1…m	Segment k
p_k, k=1…m	Split Point k; Segment 1 automatically has p_1=1
S	Minimum Segment Size
n_k	Size of Segment k
d_i, $i= p_k \ldots p_k+n_k-1$	Data Point (Row) i in Segment k
β_k	Beta Vector for Regression Model Y=[1 X]β + ε in Segment k
ε	Error Term
SSE_m	Total Sum of Squared Errors as a result of m Segments
BIC_m	Bayesian Information Criterion as a result of m Segments

The multi-objective optimization problem in a simplified form is:

$$\min: (SSE_m(p_k), m)$$
$$\text{s.t.}$$
$$m \geq 2$$
$$p_1 = 1$$
$$p_{k+1} \geq p_k + S, \forall k = 1 \ldots m-1$$
$$p_m \leq N - S + 1$$

The two objectives in this problem can be in conflict. It is straightforward to show that having the maximum number of segments can often minimize the total sum of squared errors. In fact, if the minimum segment size is 2, it is possible to construct a segmented regression model with zero errors if we allow for maximum number of segments; however, the resulting model would have no practical use. On the flip side, the minimal number of segments can be one if we do not restrict it to be at least two, but the regression model can carry many errors. In this research, the two objectives are treated in a sequential manner. First, within each segment, an OLS approach is used to construct the within-segment regression model. Then an algorithmic approach is used to determine the number of segments and split points. The computational complexity of an enumeration algorithm is in the order of $2^{\frac{n}{2}-1}$ and grows exponentially when the sample size n gets larger. It is therefore necessary to develop a fast heuristic algorithm to determine the splitting quickly and automatically.

IV. HEURISTIC ALGORITHM

Determining the number of segments and split points in TSR with multivariate time series data is computationally intractable as the problem can be reduced to a special partition problem. CART is a traditional machine learning method that can be used to partition time series data but is inefficient as a recursive algorithm. In this research we develop a fast heuristic for solving the TSR problem in a robust and efficient way. The pseudo code of the algorithm is presented in Figure 1.

Step 1:

$m = 1$

Split this segment into 2 segments:

Determine the split point by enumerating all possible split points from d_{1+s} to d_{N-s+1} using the least-squares criterion

$m = 2$

Step 2:

For Segment k, $k = 1 \ldots$ m, split the segment into 2 segments:

Determine the split point by enumerating all possible split points from d_{pk+s} to $d_{pk+nk-s}$ using the least-squares criterion

If this splitting results in an improvement in the BIC criterion ($BIC_{m+1} < BIC_m$), accept the split and $m = m+1$; otherwise reject the split

Step 3:

Stop is m is not increased; other repeat Step 2

FIGURE 1. THE HEURISTIC ALGORITHM

The algorithm uses a forward search approach that starts with the minimum possible number of segments. The termination criterion is when more splitting will not improve the Bayesian information criterion (BIC).

$$BIC_m = \log\left(\frac{SSE_m}{N}\right) + 2(m-1)\frac{\log N}{N}$$

As an example, we obtained data from the capital market. Weekly adjusted prices of a selected stock were collected from the first quarter of 2019 to the first quarter of 2023. Weekly return data was then calculated from the weekly adjusted prices. We obtained weekly return data for a selected stock, e.g., Apple (stock symbol AAPL), as well as weekly return data for a market index (S&P 500). Using the closing day of the week as the index, weekly returns of the stock as the dependent variable (y), and weekly returns of the stock market as the independent variable (x), we obtained a multi-dimensional time series data set with 169 data points.

The heuristic algorithm was implemented with Python 3.9. To validate the efficiency of the algorithm, the best segmentation was found in approximately 0.5 seconds on a Windows PC with Intel Core i7-12700H and 16GB RAM. With a minimum segment size of 5, it is best to split the AAPS returns vs. S&P500 returns data into three segments at the weeks that ended on 8/28/2020 and 5/28/2021. In Figure 2, the slope of the regression line in each of the time segment is the beta of AAPL during that period. 8/28/2020 was the date on which AAPL had a 4 to 1 split. Muna and Khaddafi (2022) showed that systematic risk often increased after the stock split activity. This positive change in the beta of AAPL was accurately captured by our algorithm. In the first half of 2021, with the news of Apple unveiling several new products and its revenue and market cap approaching record highs, AAPL's beta significantly decreases at the end of May, 2021. The reduced systematic risk ensures confidence for long-term investors.

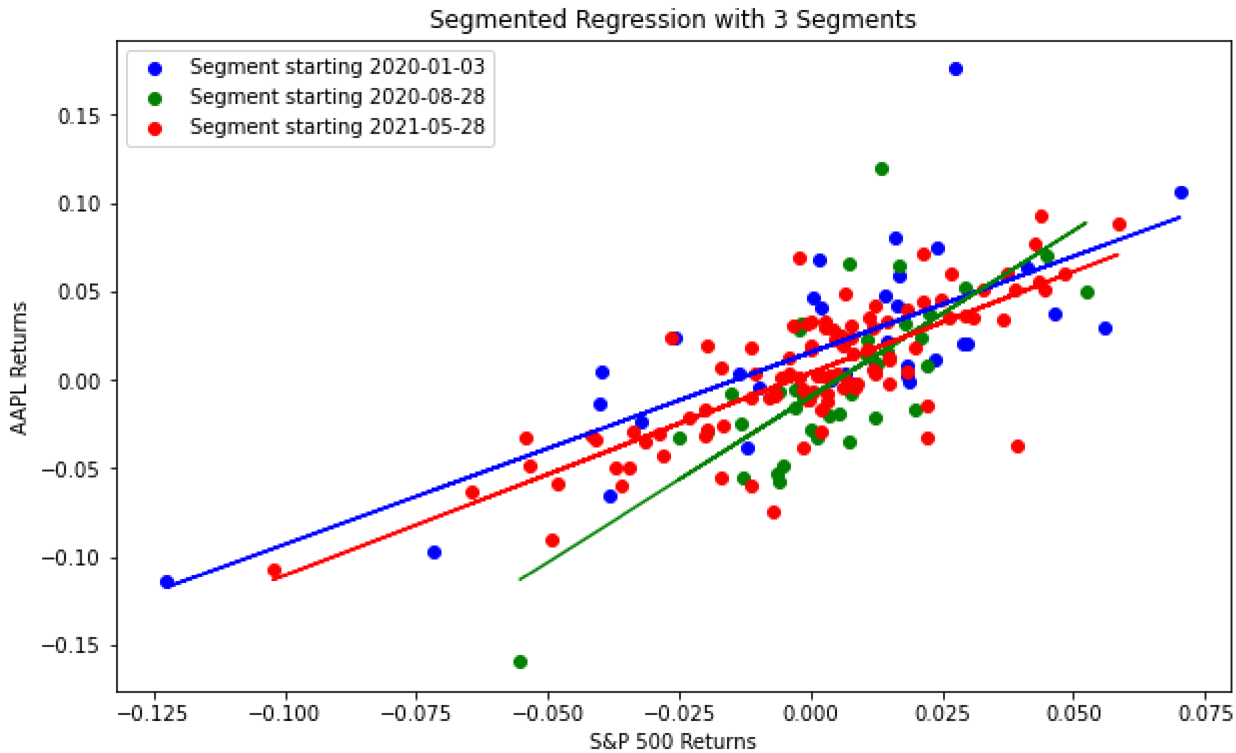

FIGURE 2. AAPL EXAMPLE

Figure 3 shows the BIC value at different number of segments (m) values when the optimal split points are obtained at each m. We show one more m setting beyond the termination criterion in this case. For the AAPL case, the best BIC is obtained at $m = 3$.

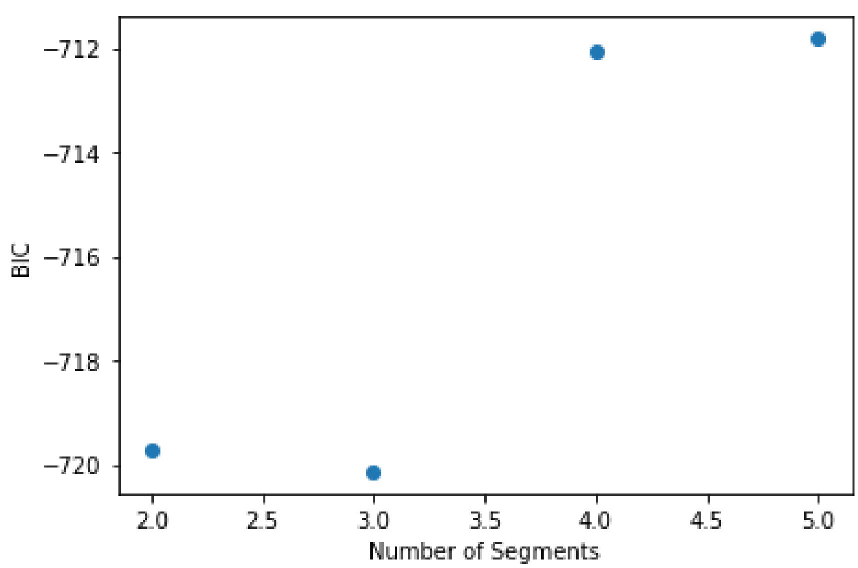

FIGURE 3. BIC VS. NUMBER OF SEGMENTS IN THE AAPL EXAMPLE

In most testing cases, we found that the search stops when the number of segments is low, which validates the efficiency of the algorithm.

V. COMPUTATIONAL CASE STUDIES

In this section, we obtained weekly return data for several stocks in the same industry. By applying the heuristic algorithm to obtain segmented regression for the return of the stock vs. the return of the market, we obtained the best set of segmentation for each stock to detect the change points of the stock beta that indicate the changes in the systematic risk of the stock. The minimum segment size is set at 5 weeks.

The first set of stock is the retail industry. This industry's operations heavily depend on the robustness of the global supply chain. Segmented regression analysis for stock betas of six of the largest retail companies in the US are shown in Figure 4. It is consistent that the systematic risk of major US retailers significantly increased around the time when the COVID-19 pandemic hit the US due to supply chain disruption as shown in their capital market data. The only exception was Lowe's, which was already suffering weaker sales pre-pandemic. On the flip side, Lowe's marketing strategy focusing on helping homeowners, rather than contractors, on home improvement projects during the pandemic when they were trapped indoors had helped the company significantly improve sales and beat earning expectations that led to lowered systematic risk. Meanwhile, Amazon benefited from the online shopping trend and was able to maintain its low systematic risk at the beginning of the pandemic; however, the beta eventually increased when the supply chain disruption persisted into the third quarter of 2021.

The second set of stocks are from the automobile industry. Ford quickly experienced an increase in its systematic risk at the beginning of the pandemic due to supply chain disruptions, particularly for semiconductor chips used in automobiles. The other large US automaker, General Motors (GM), was able to manage the chip shortage situation by diverting chips from low-sales models to high-sales models to maintain a good profitability during the pandemic. GM's increase of systematic risk did not happen until late 2021, when the chip shortage worsened. On the other hand, Tesla obtained a reduced systematic risk in the middle of the pandemic because of its popular electronic vehicles (EV). A similar trend happened to other EV makers such as Polaris. Meanwhile, chip shortage in the automotive supply chain made it difficult for consumers to buy new cars. Used car dealers such as CarMax and auto parts retailers such as AutoZone obtained a decrease in their systematic risks during the pandemic.

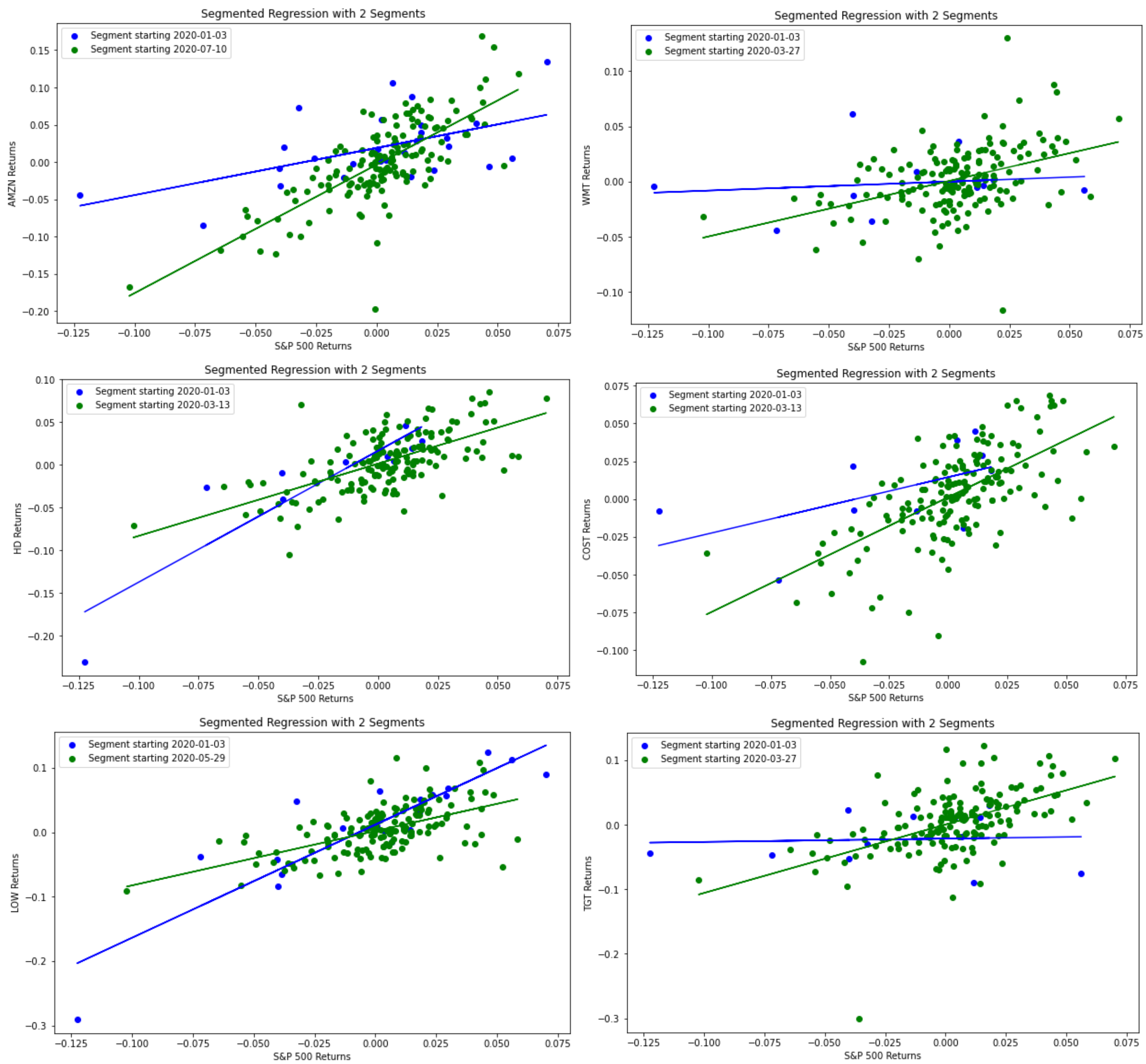

FIGURE 4. SEGMENTED REGRESSION FOR THE RETAIL INDUSTRY

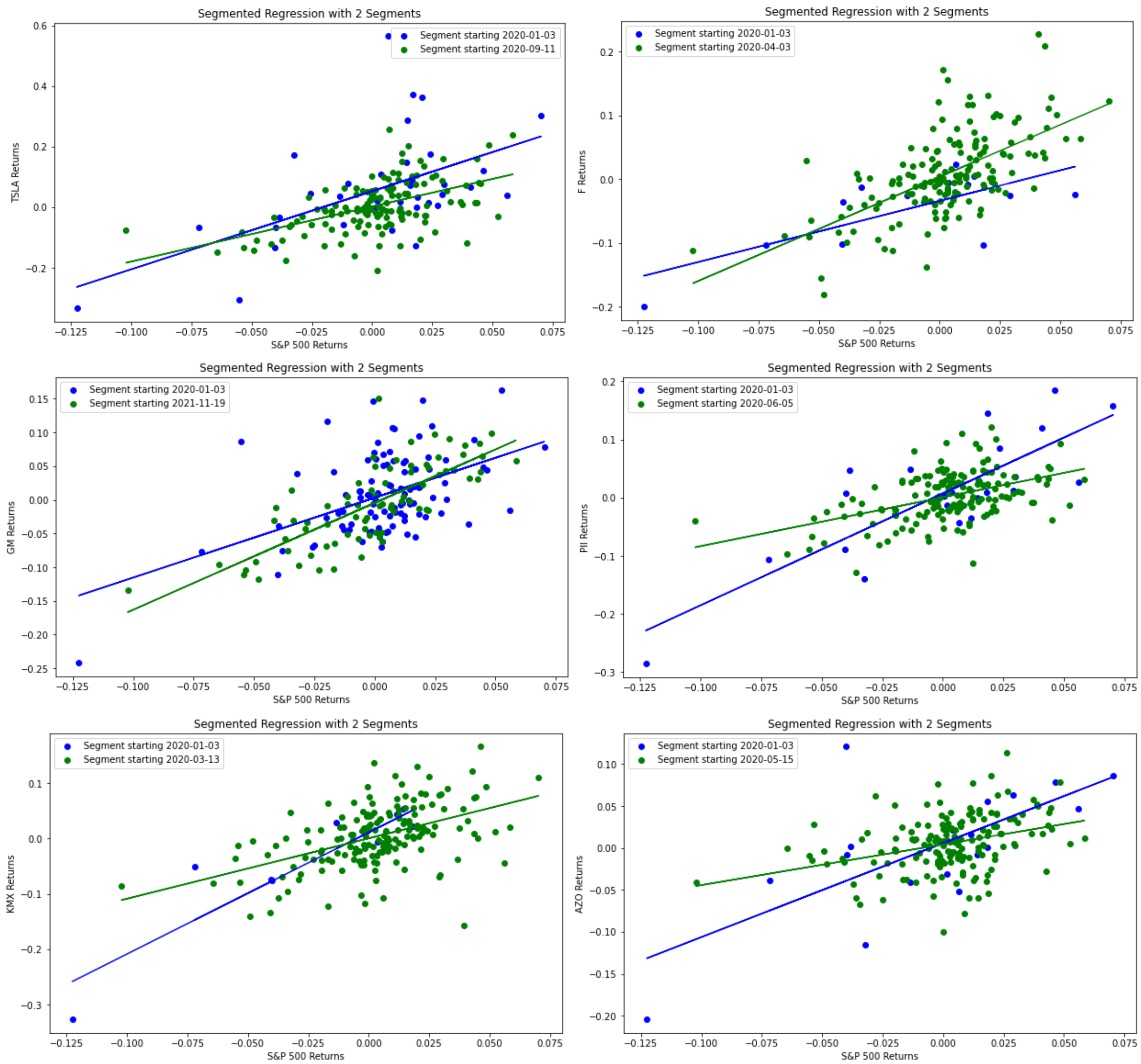

FIGURE 5. SEGMENTED REGRESSION FOR THE AUTO INDUSTRY

The third set of stocks in our analysis is from the semiconductor industry. Semiconductor manufacturing plays an important role in modern supply chains as semiconductor chips are critical to many technology-based products. Most semiconductor companies design their chips but have the chips manufactured by a third-party foundry such as TSMC or Samsung. As they compete for the capacity provided by the third-party foundries, systematic risks of their stocks increased in 2020 or 2021 as the capacity shortage worsened in the chip supply chain during the pandemic. Intel was able to maintain a stable systematic risk until the chip shortage situation improved in 2023 since Intel made chips in its own factories and did not rely on any third-party foundries. As a manufacturing service provider, TSMC was in a leading position in the semiconductor industry. Globally, over half of the semiconductor chips are made by TSMC. However, TSMC's systematic risk increased later in 2022 due to geopolitical tensions.

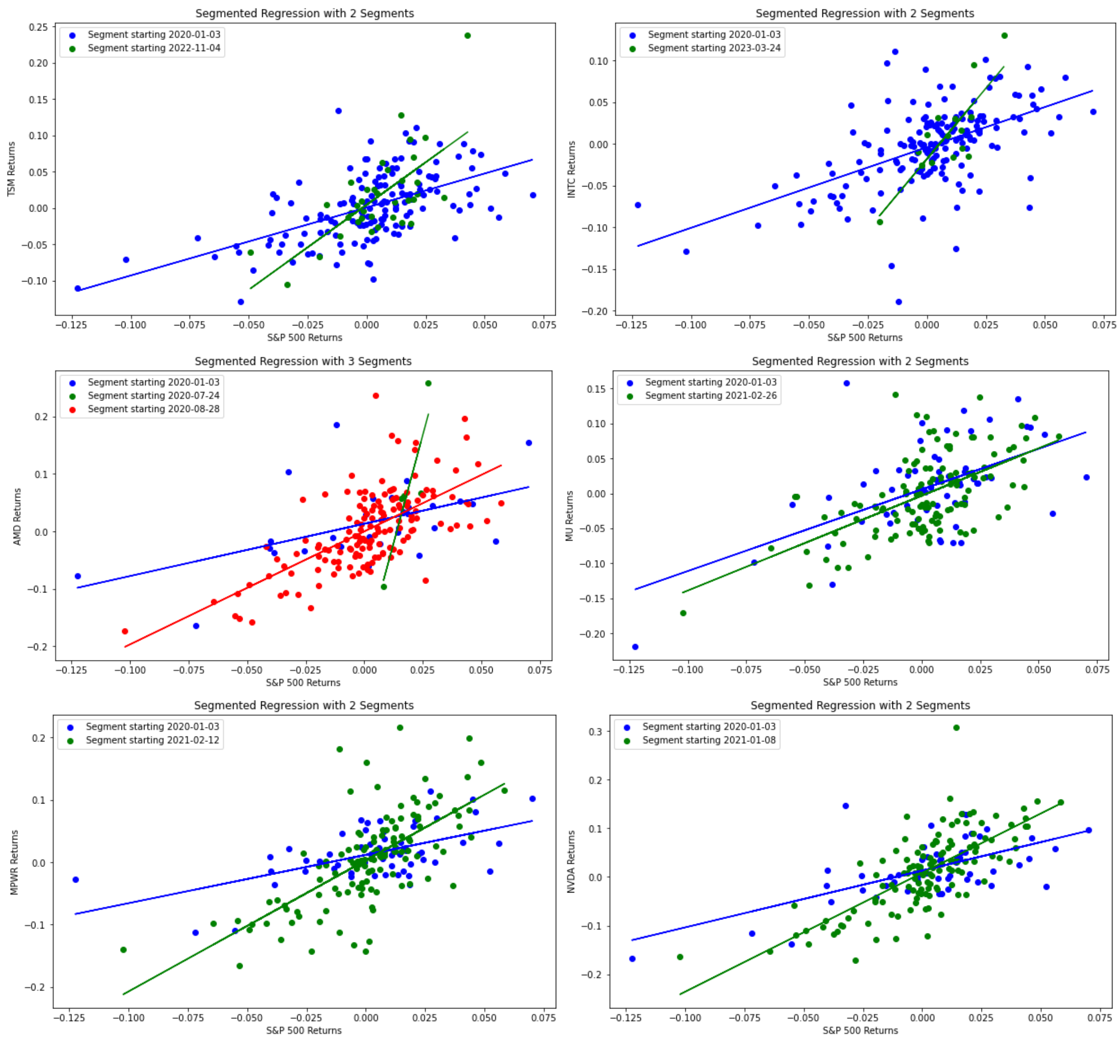

FIGURE 6. SEGMENTED REGRESSION FOR THE SEMICONDUCTOR INDUSTRY

VI. CONCLUSION REMARKS

In this research, we studied TSR problem with applications in analyzing the dynamics of stock betas. We developed a fast heuristic for obtaining the best set of time-based segments for multi-dimensional time series data with good regression fit (as driven by minimizing the sum of squired errors in regression efforts) as well as the minimum number of segments needed to detect the changes. Our case studies using stock return data since the first quarter of 2020 demonstrated that our TSR method contributes to the literature by being 1) efficient in obtaining the best segmentation for the computational intractable TSR problem, and 2) effective in detecting supply chain disruption-induced systematic risk changes for various industries. Such analysis has the potential to practically help investors and corporate executives gain a better understanding of the impact of global supply chain disruptions on businesses to develop hedging and supply chain resilience strategies. Such a tool provides insights into the direct effects of supply chain to industry adjustments on corporate end goals.

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