Risk Pooling Mitigates Leverage-Induced Principal Agent Problem

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Modern corporations are commonly managed by hired professionals. Firms and their executives may have interests that are not aligned, which leads to a Principal Agent problem in corporate governance. In this research, we show that equity holders acting as a firm's agents make aggressive operational decisions and take on excess risks. Our study establishes a positive relationship between operational aggressive decisions with a company's leverage levels. A risk pooling mechanism is recommended to resolve the Principal Agent discrepancy. The model shows that a perfect risk pooling can achieve the incentive alignments. As perfect risk pooling is hard to come by in real world, we continue to show that partial risk pooling can help mitigate leverage-induced managerial aggression in operational decisions via strategic signaling effect of debt.

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I. INTRODUCTION

Benchmark interest rates in the United States have remained low for more than a decade with a record low of 0.25 percent observed in December 2008 (Trading Economics, retrieved February 2022). Federal Reserve Chairman Jerome H. Powell (2017) commented that "low for long" interest rates can lead to excessive debt in corporate finance. In addition to a decade of near zero real interest rates, during 2008 financial crisis and 2020-2022 pandemic, the U.S. Federal Reserve (and many other central banks) carried out massive Quantitative Easing (QE) policies. Debt is more cost effective than equity financing with built-in tax advantage under very accommodating financial market. As a result, the U.S. corporate debt has increased significantly in recent 10 years and reached about $17.7 trillion at the end of 2020 (Buckley et al, 2021).

As the use of debt in a company's capital structure prevails to seek higher returns to shareholders and company values, it becomes increasingly important to understand if there are any impacts of financial leverage on business operations. A firm's leverage in the capital structure, i.e., its debt-to-equity ratio, may be set for a variety of reasons. Jensen and Meckling (1976) are among the first to demonstrate that an appropriate debt-to-equity ratio helps firms reduce agency costs. A firm's capital structure can signal its profitability to outside investors (Stiglitz and Weiss 1981). Brander and Lewis (1986) found that debt could also serve as a commitment to competition in a duopoly market. We choose to study the effect of a firm's debt on its operational decisions, rather than explain the reasons for its established capital structure, although our simulation results provide evidence that the optimal capital structure exists. Hence, we assume firms' debt levels are exogenous variables.

In practice, many firms separate their financial and production department decision processes (Wanzenried 2003). Operations management researchers generally ignore the influence of capital structure following Modigliani-Miller (1958) irrelevance propositions, which state that a firm's market valuation is independent of its financing and dividend pay-out decisions. Therefore, the propositions conclude, a firm can separate its financial and operational decisions. However, the propositions require strong assumptions that generally are not satisfied. Economics and corporate finance researchers have shown that a firm's capital structure affects its production decisions and hence its market valuation (Titman 1984; Brander and Lewis 1986). Recognizing this impact is particularly important to a firm's shareholders, managers, and competitors who seek to predict the firm's operation decisions and determine its value. Our study follows the vision promoted by Birge (2015) to study the links between the financial and operational management.

Most operational decision models maximize a firm's expected profits and recent research shows that the management team may have quite different objectives. Chen, Gilbert, and Xu (2012) argue that executives compensated primarily on stock options are more aggressive in investment and production than just optimizing the expected gains. Xu and Birge (2008) indicate that corporate debt levels have impacts on operational outcomes if managers act on behalf of equity holders. In this research, we assume a proxy management team represents the interests of the equity holders and study how to mitigate the leverage-induced principal agent problem. We assume that the firms under study adopt the limited-liability corporate principle, i.e., the firm is controlled by equity holders unless it fails to pay back its contracted debt and claims bankruptcy. If that happens, control shifts to debt holders. Other capital structures and liquidation policies may lead to different types of principal agent problems.

The mechanism we presented in this study to mitigate the leverage-induced principal agent problem is risk pooling of non-correlated demand, which has long been a tool to alleviate operational uncertainty in supply chain management (e.g., Bimpikis and Markakis, 2016; Puga and Tancrez. 2017). The idea is that a manufacturer facing high realized demand can buy final products from other companies at a spot market. The more firms participate in the trade, the higher the probability that shortages at one market can be met by surplus at other locations. An example of risk pooling platform is the cooperative electricity exchange in the power pool, such as in California (Wilson 2003) and Great Britain (Newbery 1998). Such platforms are spot markets in nature. Kleindorfer and Wu (2003) provide a survey of industrial market structure, including variables such as quantity, delivery time, or price of a spot market. Lee and Whang (2002) examine the impacts of a risk pooling secondary spot market on supply chain coordination. Mendelson and Tunca (2007) explore the strategic trading at a spot market when participants can observe and react to new information. Pei et al. (2011) investigate the role of spot markets in sourcing flexibility. Xu et al. (2020) create a new supply chain coordination vehicle by combining spot trading and quantity flexibility contracts.

Our model includes multiple local monopolies facing idiosyncratic market demand risks. Each monopoly produces a homogenous, short-lived product and bears a certain level of debt. All the firms adopt a limited-liability structure. A firm's production level may increase as it bears more debt, which is consistent with the results predicted in most limited-liability literature (Brander and Lewis 1986; Showalter 1985; Wanzenried 2003). We then investigate two setups with risk pooling arrangements. The only difference between these two scenarios is the total number of participating firms (O'Hara 1997; Kyle 1989).

In the first model, there are an infinite number of identical firms. Each firm's trading volume is small compared to the total market size. In this extreme effective risk pooling scenario, each firm produces to maximize overall firm value, and the debt level does not affect production decisions. Therefore, a fully liquid market is introduced, and the equilibrium is consistent with the Modigliani-Miller (1958) irrelevance propositions to decouple firms' financial and operational decisions.

Then we look at the situation when there are only limited number of firms available to pool demand with. Without loss of generality, we assume there are two firms trading their excess final products. The model captures a firm's strategic decisions when observing the other firm's outstanding debt. It also maintains theoretical simplicity and flexibility. We are able to show that even if the risk pooling effect is not fully effective when there are only two participants, it still reduces the aggressiveness of the decision makers in terms of production levels. In addition, the firm's production rises with its own debt, but falls as the other firm holds more debt.

We conducted simulations to analyze firms' equilibrium strategies under a variety of debt levels. Our results suggest that the firm with significantly less debt can take advantage of its counterpart's high leverage and save costs by lowering (or even ceasing) production. This is because the high-debt firm will produce more aggressively. The low-debt firm can buy from its partner and save on operational costs and reduce its financial risk. The high-debt firm, in contrast, cannot get help from its counterpart if it has a shortage. It must insure itself by increasing production. In the extreme case -- in which one firm bears enormous debt and the other has none – the risk pooling arrangements have big consequence when participants have very different leverage: The full-equity debt free firm can nearly stop producing and rely on the other firm's excess production.

The rest of the paper is organized as follows: In Section 2, we introduce the model and present our argument and major findings. In Section 3, we discuss possible extensions to and limitations of our model. The proofs are presented in the appendix.

II. MODEL AND ANALYSIS

2.1. Leverage-Induced Principal Agent Problem

In this base model, we primarily examine how a firm's financial leverage decisions and operational decisions are linked. To establish a benchmark situation, we consider N identical firms that are monopolies in their local markets. The firms produce homogenous, short-lived goods at a constant marginal cost, $c>0$, and sell them at a uniform price, $P>c$. For firm $i \in I=\{1,2,\cdots,N\}$, its local market demand, Q_i, is uncertain when it determines its production level, q_i. The firm's operational profit function is

$$\pi_i = \pi\left(q_i; Q_i, P, c\right) = P\min\left\{q_i, Q_i\right\} - cq_i \qquad \forall i = 1,...,N \tag{1}$$

Prior to realization, Q_i is regarded as an independent random variable drawn from a common distribution function, $F(\cdot)$ with support $[0,\infty)$. This means that the local markets are isolated from each other. We assume that F is twice differentiable and strictly concave.

In terms of capital structure, we assume the firms with limited liability are controlled by equity-holders. Each firm has same amount of assets and outstanding debt. Which means they have the same financial leverage. With given assets value, a firm with greater outstanding debt has higher leverage (i.e., debt to equity ratio). For our model, we denote the face value as B_i for firm i. A firm is bankrupt when its equity holders fail to pay back B_i after selling the products and the debt holders seize the control of the company assets.

Each firm makes production decisions based on its local market demand. We can hence analyze each firm separately. In firm i, equity-holder's profit is $(\pi_i - B_i)^+$. The equity holder chooses production level, q_i, to maximize their expected profit (which is different from the firm's expected gains):

$$\max_{q_i \ge 0} \mathrm{E}\left[(\pi_i - B_i)^+\right] = \int_0^\infty (\pi(q_i; P, c, Q_i) - B_i)^+ f(Q_i) dQ_i \qquad \forall\ i = 1,...,N \tag{2}$$

Lemma 1: Firm i is bankrupt when:

1. $q_i < \frac{B_i}{P-c}$; or

2. $q_i \ \ge \ \frac{B_i}{P-c}$ and the realized demand is small, i.e., $Q_i < \hat{Q}_i = \frac{cq_i + B_i}{P}$.

Note that the equity holders' profit $(\pi_i(q_i; P, c, Q_i) - B_i)^+$ is either positive or zero. Lemma 1 implies that equity holders will always produce a quantity that is more than $\frac{B_i}{P-c}$. We can thus regard $\frac{B_i}{P-c}$ as the lower bound of the firm's production. It increases with the firm's debt level / leverage. According to Lemma 1, we can rewrite the equity-holder's optimization problem as

$$\max_{q_i \ge \frac{B_i}{P-c}} \mathrm{E}\left[\pi_i - B_i\right] = \int_{\hat{Q}_i}^{q_i} (PQ_i - cq_i - B_i) f(Q_i) dQ_i + \int_{q_i}^{\infty} (Pq_i - cq_i - B_i) f(Q_i) dQ_i \qquad \forall\ i = 1,...,N \tag{3}$$

In order to rule out extreme equilibrium cases such as zero or infinite production levels, we must impose two more assumptions. First, we assume that $F\left(\frac{B_i}{P-c}\right) < 1$, which guarantees that the debt is not impossible to repay. Hence, the firms will be willing to produce. Second, we assume that $T(x) = \frac{-f'(x)}{f(x)}$ is non-decreasing in x. This ensures that the probability of extremely high demand decreases fast enough to keep production from reaching infinity. Denote q_i^{NO} as the optimal production level for firm *i's equity holders* when firms are completely independent of each other without any exchange of output. Proposition 1 characterizes q_i^{NO}.

Proposition 1: For any debt level B_i, q_i^{NO} uniquely exists, and $q_i^{NO} > \frac{B_i}{P-c}$.

Proposition 1 enables us to discuss the impacts of debt levels on the operational decisions of limited-liability firms and compare that to other cases. The result is summarized in Proposition 2.

Proposition 2: The optimal production, q_i^{NO}, increases in the debt level i.e., $\frac{\partial q_i^{NO}}{\partial B_i} > 0$.

Proposition 2 shows that, in equilibrium, firm i increases production as it bears more debt. In traditional operations decision models, we attempt to maximize the expected profit under all demand realization value as all the loss and gains are financial consequences to the firm. However, for a firm following limited liability principle, the equity holders' downward risks are curtained, and they make more aggressive operational decisions, such as higher output levels. The implication from Proposition 2 is that with a elevated debt level, the equity holders becomes more aggressive in operations.

Proposition 2 also implies a firm's fundamental corporate structure leads to different output levels. As the optimal production increases in the debt level, a debt free firm has lower optimal production quantity than debt bearing firms under limited liability principle. It is easy to show that organizations running without limited liability principle is equivalent to a debt free firm in terms of production decisions. Thus, an unlimited liability structured firm makes conservative operational decisions. We summarize this result in Lemma 2. Recall that q^{NO} is the optimal production quantity for firms under limited liability. Now we denote the optimal production levels for firms without limited liability as q^{NL}. We now have the following Lemma 2.

Lemma 2. $q^{NO} > q^{NL}$

The first order derivative of (3) can be written as

$$\left\{P\left[1-F\left(q_i\right)\right]-c\right\}+cF\left(\hat{Q}_i\right) \qquad (4)$$

The part in parentheses is identical to the one in the case without limited liability. The extra term in (3.4), $cF\left(\hat{Q}_i\right)$, represents the impact of limited liability when the firm includes a positive debt level in its capital structure. Since $cF\left(\hat{Q}_i\right)$ is strictly positive, we conclude that the limited-liability firm will choose a higher production level. We call this over-production the "Leverage-Induced Aggression Effect," which is the consequence of non-aligned incentives between a firm and its equity holders.

2.2. Mitigate of Leverage-Induced Principal Agent Problem Using Risk Pooling

In our benchmark base model, we proved that the Leverage-Induced Aggression Effect leads to elevated operational activities in companies that are independent to each other. In this section, we show that a risk pooling ecosystem, where excess output can be sent to another market, can reduce the excessive risk-seeking behaviors caused by company financial leverage and thus fully resolve the principal agent problem.

The timeline is illustrated in figure 1. With the given debt levels, equity holders of firms make production decisions simultaneously before the demand is realized. After satisfying local demand, each firm simultaneously submits its trade plan to sell its remaining inventory. Without knowing the realization of demand for other firms, each firm's quantity to exchange is $|q_i - Q_i|$. The market-clearing price, *w*, is decided by the total supply and demand in the trade.

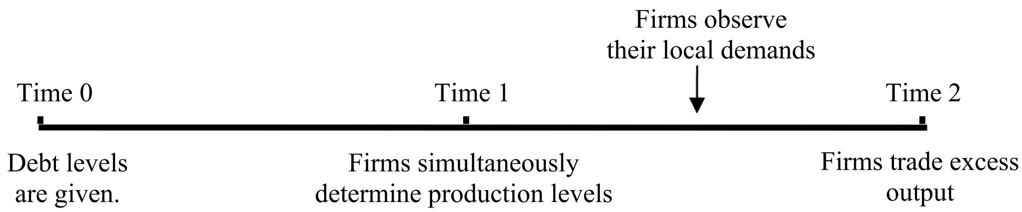


FIGURE 1: TIMELINE.

Note that we assume the debt levels are public information, which is relatively close to reality for publicly traded companies. However, for private firms, the financial statements may not be easily available or observable. The purpose of this work is to link firms' financial and operational decisions and future research can be conducted to examine the market dynamics when the debt level is not fully transparent.

We consider an ideal case that includes a very large number of identical firms. As a result, each firm's production has a negligible impact on the market price. By the Law of Large Numbers, we have

$$\lim_{N\to\infty} \overline{Q} = EQ. \quad (5)$$

Since the firms bear identical debt ex ante, we focus on the symmetric equilibrium. We suppress the subscript *i* for each firm and use superscript FL to denote the case with a large number of market participants. The equilibrium is summarized in Proposition 3.

Proposition 3: When $N \to \infty$,

1. the market clearing price $w = c$;
2. the equilibrium production is $q^{FL} = EQ$;
3. and $\partial q^{FL} / \partial B = 0$.

Proposition 3 shows that the market-clearing price is equal to the marginal cost of production. In addition, each firm's production level is the same as the expected demand in the equilibrium. Because of the stability of the average market demand, the firms can fully hedge their market risks. The market-clearing price will not deviate from the marginal cost so to avoid over-supply or shortage. The critical assumption is that each firm's trading volume is tiny compared to the total market size. No one firm expects to affect the market price, *w*, through its own order and, therefore, the exchange market achieves full liquidity. Number 3 of Proposition 3 shows that a firm's debt has no impact on its production choice; production and financial decisions are unbundled under effective risk pooling.

This model depends on the assumption of an infinite number of small firms. In the next section, we determine if the production and financial decisions will still be separated when perfect risk pooling is impossible.

2.3. Risk Pooling Mitigates Leverage-Induced Agent Aggression in a Two-Company Setting

In this section, we keep the same model setup, except that N =2, and derive the market equilibrium. We denote the two firms as *i* and *j*. The firms' strategies are captured by their production decisions, (q_i, q_j). We solve the equilibrium strategy, (q_i^*, q_j^*), by backward induction.

2.3.1. Trade Platform Clearing Price, w

One of six cases is possible after the firms make production decisions and realize the demand in their local market. These cases are summarized in Table 1:

TABLE 1. THE CASES AFTER DEMAND REALIZATION.

Case 1	$Q_i - q_i > q_j - Q_j > 0$
Case 2	$q_j - Q_j > Q_i - q_i > 0$
Case 3	$Q_j - q_j > q_i - Q > 0$
Case 4	$q_i - Q_i > Q_j - q_j > 0$
Case 5	$Q_i - q_i > 0$ & $Q_j - q_j > 0$
Case 6	$Q_i - q_i < 0$ & $Q_j - q_j < 0$

The two firms only exchange if one has excess products and the other has a shortage (Cases 1 through 4). The trade is not possible if both firms have excess or shortage simultaneously (Cases 5 and 6).

In Cases 1 through 4, the exchange demand, $D(w)$, for the buying firm, i (without loss of generality) is

$$D(w) = \begin{cases} Q_i - q_i & if \quad 0 \le w < P \\ [0, Q_i - q_i] & if \quad w = P \\ 0 & if \quad w > P \end{cases} \qquad \forall i = 1,2 \tag{6}$$

Since the firm has local shortage, it wants to buy as much as possible when the price is lower than *P*. However, when the market-clearing price is *P*, the firm is indifferent toward buying. The firm will stop buying when the price rises above *P*. The selling firm's supply function is

$$S(w) = \begin{cases} q_j & if \quad w > P \\ q_j - Q_j & if \quad 0 \le w \le P \\ [0, q_j - Q_j] & if \quad w = 0 \end{cases} \quad \forall j = 1,2 \tag{7}$$

The firm is willing to sell at any positive price because it already paid the cost of production. There is no extra local demand and selling to the other firm is the only way to earn extra money. The firm is indifferent toward selling when the market-clearing price is zero. Given the firms' trading strategies, the market-clearing price, *w,* is defined as

$$w = \begin{cases} P & If \ Q_i - q_i \ge q_j - Q_j \\ 0 & If \ Q_i - q_i < q_j - Q_j \end{cases}$$
$$\forall i, j = 1,2 \tag{8}$$

A simple calculation shows that

Lemma 3. $\frac{dE_{Q_i,Q_j} w}{dq_i} < 0$

Lemma 3 illustrates that in the two-firm model, the market-clearing price is influenced by the firms' production strategies (q_i, q_j). The expected market-clearing price, $E_{Q_i,Q_j} w$, is

$$E_{Q_i,Q_j} w = P \cdot Pr\{(Q_i + Q_j) \ge (q_i + q_j)\} \tag{9}$$

Given firms' local demands (Q_i, Q_j), firm i expects that market-clearing price decreases with the production level, since the increase in the production lowers the chance of the market price being high.

2.3.2. The Equilibrium Production Strategies.

We now determine the firms' production decisions in period 1. A firm selects a production level to maximize its equity-holders' expected profits. Decision-makers know that the profit is contingent on the other firm's production decision and the possible realization of demand in both local markets. Depending on range that the optimal solution falls into, firm i's expected profit takes two forms:

$$\underset{q_i}{Max}\ \ G_i = \begin{cases} G_i^1\left(q_i;q_j,B_i\right) & q_i < \dfrac{B_i}{P-c} \\ G_i^2\left(q_i;q_j,B_i\right) & q_i \geq \dfrac{B_i}{P-c} \end{cases} \quad (10)$$

Denoting the probability of bankruptcy in the cases with and without the trading opportunity as $\Gamma(Q_i,Q_j)$ and $F(Q_i)$, respectively, yields

Lemma 4: $\Gamma(Q_i,Q_j) < F(Q_i)$

Recall that Lemma 1 shows that, without trading opportunities, a firm always goes bankrupt when it produces less than $\frac{B_i}{P-c}$. Equation (10) implies that a firm with an output trading partner may still expect a strictly positive profit even if it produces less than $\frac{B_i}{P-c}$. This is because the firm has a chance to buy products from the other firm. The Lemma 4 also shows that a firm with a B2B selling opportunity is less likely to go bankrupt if it chooses a production level more than $\frac{B_i}{P-c}$.

2.3.3. The Numerical Analysis

We next determine the optimal production level for each firm. Unfortunately, the objective function is complex and non-linear; its properties can be analyzed only for an explicit distribution. Consider the exponential distribution. The demand distribution is now represented as $f(Q_i) = \lambda e^{-\lambda Q_i}$; $\forall\ i = 1,2$. The exponential distribution follows the characteristics of the distribution defined earlier in this paper. We assume that $\frac{1}{\lambda} > \frac{B_i}{P-c}$. $\forall\ i = 1,2$, which guarantees both that the expected demand is not too low and that the probability of demand will not fade out too quickly.

- **Symmetric Case**

In this section, we only consider the optimal solution: When the two firms have debt with identical face values. $B_i = B_j = B$. We are only interested in symmetric, sub-game, perfect equilibrium ($q_i^{EX} = q_j^{EX} = q^{EX}$) here.

Lemma 5: If there exists a symmetric equilibrium such that $q_i^{EX} = q_j^{EX} = q^{EX}$, then $q^{EX} > \frac{B}{P-c}$ must hold.

The two firms cannot simultaneously choose production levels lower than $\frac{B}{P-c}$ in the equilibrium. This is because a firm will only scale back production if it knows that the other firm will produce enough for both of them. Since Lemma 3 rules out the existence of a symmetric equilibrium in Case 1, we focus only on Case 2. The first order derivative from Case 2 is

$$\begin{aligned} &(Pe^{-\lambda(q_i+q_j)})\left[1+\lambda(q_i+q_j)-\lambda^2\left(q_i+q_j\right)q_i+\lambda^2\frac{1}{2}\left(q_i+q_j\right)^2\right]-c \\ &+e^{-\lambda\left(q_i+q_j\right)}\frac{P\lambda^2}{2}\left(\hat{Q}_i+\frac{2c}{P\lambda}\right)\hat{Q}_i+c\left[1-e^{-\lambda\hat{Q}_i}\right] \end{aligned} \quad (11)$$

Proposition 4: The optimal production q^{EX} uniquely exists.

Equation (11) shows the existence of equilibrium. In addition, if we denote that q^{NL} is the optimal production level in the case when a firm is not built upon limited liability principle, then the result can be summarized as in Lemma 6.

Lemma 6: q^{EX} is greater than q^{NL}.

As with Lemma 2, we can prove that the first two terms of (11) are identical to the first-order derivative of the profit maximization problem in the case without limited liability. Therefore, the last two terms of (11) represent the limited-liability effect. Since the sum of the last two terms is always positive -- even when the firms can exchange in the market -- the limited-liability effect still leads to over-production.

Although Proposition 4 presents the existence and uniqueness of the optimal solution under a general condition, it is still unlikely that we can derive a closed-form solution. This is because the functional forms of the objective function and the first-order condition are complex. But a general analytical solution is unnecessary because we seek only to justify the possibility relationship between a firm's capital structure and its operational decisions when its operational risks are reduced. Rather than performing a complicated mathematical proof in the next section, we use the simulation to show the comparative static analysis results.

We next compare the equilibrium of the current two-company model with and without opportunities for risk pooling (P = 1 through 100). The combinations of P, c and B were randomly chosen under the constraint $\frac{1}{\lambda} > \frac{B}{P-c}$ in order to follow the assumption in the model. We present simulation results and numerical examples in the next section.

Result 1. $\frac{\partial q_i^{EX}}{\partial B_i} > 0$ and $\frac{\partial q_i^{EX}}{\partial B_i} > \frac{\partial q_i^{NO}}{\partial B_i}$ $for\ i = 1, 2$

TABLE 2.THE EFFECT OF THE FIRM'S OWN DEBT.

	Without Risk Pooling					With Risk Pooling				
	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
$P=10$	0.200	0.200	0.200	0.200	0.200	0.302	0.315	0.324	0.339	0.355
$P=20$	0.100	0.100	0.100	0.100	0.100	0.148	0.151	0.154	0.157	0.161
$P=30$	0.067	0.067	0.067	0.067	0.067	0.098	0.099	0.101	0.102	0.103
$P=40$	0.050	0.050	0.050	0.050	0.050	0.073	0.074	0.075	0.075	0.076
$P=50$	0.039	0.039	0.039	0.039	0.039	0.058	0.059	0.059	0.060	0.060

TABLE 3.THE EFFECT OF THE OTHER FIRM'S DEBT.

	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
$P=10$	-0.185	-0.197	-0.207	-0.221	-0.237
$P=20$	-0.090	-0.093	-0.096	-0.099	-0.102
$P=30$	-0.059	-0.060	-0.061	-0.063	-0.064
$P=40$	-0.044	-0.045	-0.045	-0.046	-0.047
$P=50$	-0.035	-0.035	-0.036	-0.037	-0.037

TABLE 4.THE OPTIMAL PRODUCTION.

	Without Risk Pooling					With Risk Pooling				
	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$	$B=1$	$B=2$	$B=3$	$B=4$	$B=5$
$P=10$	2.97	3.17	3.37	3.57	3.77	2.70	2.82	2.93	3.05	3.17
$P=20$	2.87	2.97	3.07	3.17	3.27	2.64	2.70	2.76	2.82	2.88
$P=30$	2.84	2.90	2.97	3.03	3.10	2.62	2.66	2.70	2.74	2.78
$P=40$	2.82	2.87	2.92	2.97	3.02	2.61	2.64	2.67	2.70	2.73
$P=50$	2.81	2.85	2.89	2.93	2.97	2.61	2.63	2.65	2.68	2.70

Result 1 confirms the existence of Leverage-Induced Aggression in operational decisions, i.e., production increases when the firm has more debt. More importantly, the results show more severe aggressiveness in operations under partial risk pooling. This is the core argument in this paper: Even if firms can reduce their risks through risk pooling, the effect of debt induced higher production is likely greater than when two firms are completely independent of each other.

However, a firm's equilibrium production level is also a result of its strategic interactions with the trading partner. We summarize the effect of the partner's debt on a firm's production decisions in Result 2 and present part of our findings in Table 3.

Result 2. $\frac{\partial q_i^{EX}}{\partial B_j} < 0 \quad for\ i \neq j$

Result 2 and Table 3 show that one firm's optimal production level is negatively correlated with the other firm's debt. In other words, the higher one firm's debt, the less quantity the other company produces. Thus, a firm's high leverage leads to the other firm's less aggressive operational decisions.

According to Results 1 and 2, a firm's debt level encourages higher output from its own operations but signals the other firm to reduce the production. It is of importance to see which force dominates. The optimal output level from Result 3 and Table 4 is the equilibrium of the strategic interaction between two firms.

Result 3: $q^{NO} > q^{EX} \quad for\ i = 1,2$.

Results 3 shows that equilibrium output is lower when firm I has an opportunity to sell or buy from firm j and takes advantage of the risk pooling of such arrangements. The overall results indicate a mitigation effect of risk pooling on the Leverage-Induced Operational Regression. The debt held by firm j indicates an expected higher level of output from firm j. That higher level of output from the trading partner is equivalent to potential safety stock to firm i. As a result, firm I produces a smaller quantity and is less influenced by Leverage-Induced Operational Aggression.

Equation (12) shows that firm i's marginal profit decreases as firm j increases production.

$$\frac{\partial^2 \pi}{\partial q_i \partial q_j} = \frac{\partial FOC_i}{\partial q_j} \overset{q_i = q_j}{=} -\lambda e^{-\lambda(q_i+q_j)} \left\{ \lambda \left(Pq_i - c\frac{cq_i + B_i}{P} \right) + \frac{1}{2}\lambda^2 P \left(\frac{cq_i + B_i}{P} \right)^2 \right\} < 0 \quad for\ i \neq j \tag{12}$$

This result is because the expected market-clearing price increases as firm j's production drops. Firm I benefits from increasing production because it may have a better chance to sell its excess at price P.

- **The Asymmetric Case**

We now assume that the two firms have different capital structures and $B_i > B_j$ $for\ i \neq j$. In Figure 2, we compare the equilibrium between the symmetric case and the asymmetric case (in which we assume $B_i = 2B_j$).

FIGURE 2. THE BEST RESPONSE OF TWO FIRMS.

(Left graph: $B_i = 50, B_j = 50$. Right graph: $B_i = 50, B_j = 25$)

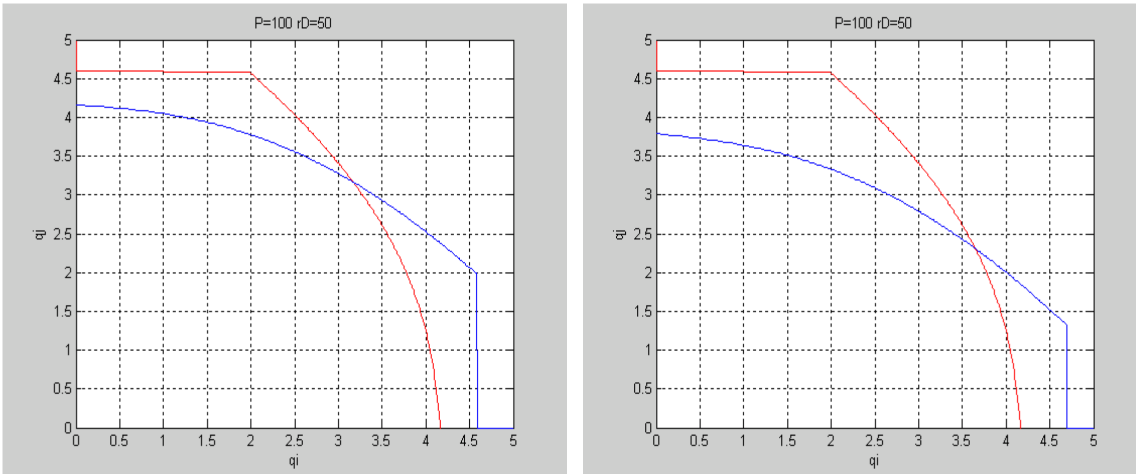


TABLE 5. THE OPTIMAL OUTPUT (P=100).

		$B_i = 10$	$B_i = 20$	$B_i = 30$	$B_i = 40$	$B_i = 50$
$B_j = B_i / 2$	q_i	2.79	3.08	3.36	3.63	3.88
	q_j	2.55	2.35	2.14	1.92	1.70

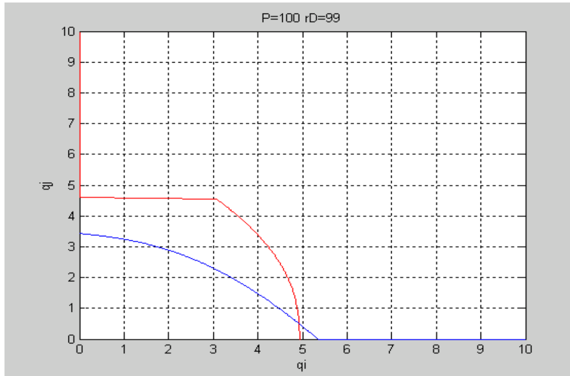
According to Figure 2:

Result 4: If $B_i > B_j$, then $q_i^* > q_j^*$.

Everything else being equal, the more leveraged firm is more aggressive in making more products. This result is consistent with the Leverage-Induced Operational Aggression. Furthermore, firm j observes higher leverage held at firm i, which is a signal of higher output, and thus will lower its production accordingly.

We argued that in the symmetric model an equilibrium cannot exist when $q_i < \frac{B_i}{P-c}$ because both firms have the same ex ante obligation and so should behave the same way. This conclusion may not be true in the asymmetric case. Consider an interesting case in which one firm has zero debt and the other firm bears a very high level of debt ($B_i > 0, B_j = 0$). The debt free firm has the least incentive to produce. Hence, the debt-laden firm predicts that it will get little help fulfilling any shortage from the other firm. Meanwhile, there is a fairly good chance for the debt-laden firm to profit from the trade when the debt free firm's local demand turns out to be high. Therefore, two forces drive the debt-laden firm to produce at a much higher level. Figure 3 shows that when a debt-laden firm produces aggressively while the optimal solution for the debt free firm is close to zero.

FIGURE 3. THE BEST RESPONSE.
($B_i = 99, B_j = 0$ **)**



In this situation, the debt-laden firm commits to very high production in order to pay back its debt. Since the full-equity debt free firm has a zero lower bound for production, it can simply shut down its machines and purchase from the debt-laden firm.

III. CONCLUSION

Our paper established the case that a firm's financial decision may create and amplify the incentive discrepancy between a firm and its executives. When a firm increasingly depends on debt financing, the equity holders, who typically have direct or indirect control of the firm's operations, becomes more aggressive in production activities. Given U.S. corporate debt is at the historical record high level, we expect to observe an extremely volatile business environment, with outsized success and tremendous disruptions.

The principal agent problem in our model is rooted in limited liability corporate principle, which is well studied in economics literature (Brander & Lewis 1986; Wanzenried 2003). We expect to see principal agent problems in other corporate structures, which could be the topic of our future research.

Researchers examine the effect of financial leverage in other issues has argued that the effect of capital structure is less powerful when the firm can better hedge its operational risks. (Froot et al 1993; Spano 2004). Our research supports their conclusion from the operational perspective of business. As risk pooling is known to reduce loss linked to demand uncertainty, we proposed firms to participate in trading excess output agreements with firms exposed to different market and economic conditions. Our model shows that under perfect risk pooling, firms produce at the level that maximize the expected gains, which means the leverage-induced principal agent problem is fully resolved. In such situation, firms can separate their financial and operational decisions and optimal outcomes are achieved at equilibrium. For future research, we propose to examine the effect of other operational risk reduction mechanisms to prevent the incentive frictions between the firm and its executives, such as lead time deduction, postponed customization, and collaborated forecasting and planning.

In real business world, most likely, there are limited number of firms available in post market trade and thus perfect risk pooling is hard to achieve. There is a chance that the realization of firms' demand turns out to be in the similar condition, i.e., all firms have excess supply or shortage, even if firms have idiosyncratic demand risks. In traditional economic or financial research, this "macro"-like shock is usually caused by a perfectly correlated risk faced by all firms in ex-post market (Persaud, 2003). Recent examples include the 2008 global financial crisis and the recent Covid-19 pandemic, both created synchronized shocks across the planet. However, in this paper, we have showed that even with idiosyncratic shocks, this could happen, and the chance could be large especially when we deviate from the conventional assumption of fat tail distribution of demand risk and consider extreme events as ancillaries (Taleb 2001).

However, even with limited number of risk pooling partners, the trading arrangements provide a mechanism for strategic moves upon observing other firm's debt. Our result shows that high leverage at a firm act as a signal to other firms to lower the outputs, thus the aggressiveness in over production is partially mitigated when we take all firms as a whole. This is an important takeaway from this research that even if operational risk reduction mechanisms fail to achieve full effectiveness, as long as participants are linked, they will behave strategically on the observation of each other's debt level. Such strategical signaling game leads to an overall improvement of firms' operational levels.

As higher leverage signals higher operational activities, the observable debt level can be used in supply chain to coordinate investment, inventory, and production commitments. We suggest further study of the strategic roles of a manufacturer's debt level in coordinating supply chains.

Another interesting aspect that is worth exploring in the future research is the impact of operation levels on firms' financial decisions. For example, during the pandemic, firms experienced longer lead time, which may lead to higher order quantity that demands more working capital. In addition, our research shows that a leveraged firm tends to produce more, which means the firm may need more financial resources and take on more debt. The link between a firm's financial and operational decisions may be better described as a self-reinforced feedback loop.

Our production levels maximize equity holders' gains under uncertain demand. Other factors impacting the production levels include production capacity, inventory, lead time, batch size, supply constraints, and many more. These factors have been discussed in varies modeling and empirical studies in supply chain and operations management research. We suggest including these factors into the models for future research.

Finally, further research could work on more comprehensive models that incorporate more relaxed assumptions. For example, we assume zero bankrupt costs for the simplicity of mathematical illustration. In practice, a bankruptcy imposes extra costs such as financial distress, inefficient liquidation, and managerial reputation loss. If we add an emotional loss associated with bankrupt in our model, it is reasonable to predict that equity holders would make conservative production decisions, which is the opposite of our results. However, we believe a risk pooling strategy can still serve as a tool to mitigate such under-production tendency caused by intangibles bankruptcy costs.

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