

# Optimal Video Content Production and Advertising Strategies

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As content creation becomes increasingly professionalized, video creators face critical decisions about resource allocation and production strategy. This study examines how key factors - including audience engagement metrics, creator authority, production value, operational costs, and marketing investments - influence the profitability of video content over time. Using a two-period model, we analyze the financial implications of different production approaches (in-house production vs. professional outsourcing) and marketing strategies. Our findings indicate that when initial marketing costs are below a certain threshold, maintaining in-house production is more profitable across both periods. However, as marketing costs increase, there exists a critical point that determines whether transitioning to outsourced production in the second period becomes optimal.

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## I. INTRODUCTION

With the development of the internet and social media, video has become a versatile and dynamic medium permeating almost every facet of our modern lives. Especially during the COVID-19 pandemic starting in 2020, the influence of videos has grown exponentially, with a dramatic increase in the number of users of many online video platforms. According to Cohen (2022) on the YouTube Official Blog, YouTube has surpassed 80 million Music and Premium subscribers globally, a 30 million increase from 50 million last year. As a new form of media, video is becoming increasingly

indispensable in today's digital age. Another investigation of Statista verifies this statement. According to a recent report, there was an explosive growth of daily active users on YouTube starting in 2020 Q1, which corresponds to the starting time of the COVID-19 pandemic (Ceci, 2023).

Many factors contribute to the significant audience growth in the online video industry. One of the primary advantages of video lies in its capacity to effectively communicate complex ideas and information. Videos combine visual and auditory elements, allowing for a multisensory experience that can convey emotions, nuances, and context in a way

that text or images alone cannot. In educational settings, this is invaluable, as it provides a more engaging learning experience. The use of video in classrooms has enabled teachers to create interactive lessons, making abstract concepts tangible and fostering a more profound understanding among students. Additionally, video has become a pivotal tool of political advertising. Recent U.S. presidential campaigns have leveraged its potential to reach a vast audience by strategically incorporating online videos to enhance their outreach and engagement efforts (Colvin, 2023).

Our study employs optimization models to analyze the profitability of a content creator (he thereafter) in his video production and advertising decisions on an online platform. The video creation spans two periods, wherein the content creator divides a long series of videos into "period 1" and "period 2" segments and publishes them separately. Dividing long videos into multiple segments is a common practice on platforms like YouTube. Seidel (2024) analyzed 237,928 learning videos on YouTube and found that approximately 8% of short videos and 16% of medium-length videos were part of a segmented series. This approach offers several benefits for content creators, as it allows for optimizing audience engagement, content discoverability, and revenue generation (New Image Media, 2023). For example, YouTube favors consistency in uploads and often recommend videos from the same creator. By splitting content into smaller series, creators increase the chances of being featured in recommendations. In addition, splitting content into smaller, exclusive series encourages subscribers to support for access to future episodes and improves creators' revenue.

The quality of the videos in the first period will impact the viewership of the same series of videos in the second period. Specifically, if the quality of video is lower than the expected quality in the first period, then viewers will be disappointed, resulting in less viewership in the second period. Content

creators have two ways to create the videos: outsource the production to professionals with a fee or self-edit the videos with no cost. It is reasonable to assume that the quality of the outsourced videos is higher than that of the self-edited ones; otherwise, there would be no justification for outsourcing due to its higher cost. The quality of a video is positively correlated to video view rate, indicating that higher quality videos generate higher earnings.

The content creator can invest in advertising to improve the visibility of his videos. Many video platforms, like YouTube, directly offer official promotion/advertising functions (YouTube, 2025). Advertising from third-party websites are also common practices. For example, a third-party website of TikTok provides a service of 2000 views on one TikTok video for 2.5 dollars (Celebian, 2025). It also provides TikTok likes service, for instance, a blogger can pay around ten dollars for 1000 TikTok likes (SocialBoss, 2025). The TikTok likes can also improve the visibility of videos. In this study, we aim to address the following research questions:

*RQ1: Under what conditions does in-house production yield optimal profitability compared to professional outsourcing? What market and operational factors determine the superiority of each production strategy?*

*RQ2: What is the optimal marketing investment strategy for content creators, and how does marketing spend influence the choice between in-house production and outsourcing across different periods?*

We formulate a two-period analytical model to investigate these research questions while considering various important factors in online content creation. Our theoretical analysis reveals systematic patterns in how content creators should adjust their advertising strategies based on multiple factors. These include production quality levels, advertising costs in both periods, reputation effects, audience disappointment levels, and advertising effectiveness. The major contribution of our

study is that we provide a comprehensive decision framework with clear rules based on quantifiable thresholds. This framework provides content creators optimal strategy choices and practical decision paths based on their specific circumstances, which include advertising thresholds, quality requirements, and audience expectations.

## **II. LITERATURE REVIEW**

This research is closely related to a growing body of literature exploring video content optimization and dissemination strategies. Valeiras-Jurado & Bernad-Mechó (2022) analyze modal density and coherence across various video genres, revealing distinct developmental stages in video content creation. This research provides an understanding of how genre-specific characteristics influence viewer perception and content effectiveness, offering valuable guidance for tailoring video dissemination strategies to different audience preferences.

Many studies on video optimization and audience retention are in the field of education, due to the fast development of online education and distance learning. Lijo et al. (2024) evaluate the performance of a STEM YouTube channel over a six-year period, finding that video length plays a pivotal role in optimizing audience retention. Their analysis of metrics such as views, view duration, and likes highlights the importance of content length in engaging viewers. Additionally, Lijo et al. (2024) suggest that STEM channels with pedagogical goals can enhance their reach and influence by increasing their focus on educational purposes, which indicates the potential for targeted content strategies in online education. Girón-García & Fortanet-Gómez (2023) investigate the selection criteria employed by English for Special Purposes (ESP) educators when choosing digital genre videos for classroom use. Their work explores the characteristics that make videos effective for fostering student

engagement in ESP education, such as relevance to specialized topics, clarity of presentation, and alignment with learning objectives. They emphasize the pedagogical value of well-curated digital content and provides a foundation for understanding how video selection impacts learning outcomes.

Another stream of literature focuses on video recommender systems (VRS). Lubos et al. (2023) presents a comprehensive overview of the current state of VRS, exploring the algorithms used, their applications, and the challenges faced in the field. It provides insights into the evolution of VRS and discusses future directions for research and development. Lai et al. (2024) discuss the development of recommender systems that provide personalized suggestions by analyzing user preferences. It highlights the success of such systems across various applications and emphasizes the importance of mining user behavior data to enhance recommendation accuracy. Wang & Zhao (2022) conduct a comprehensive survey exploring the development and application of affective video recommender systems (AVRS). The AVRS integrate multidisciplinary approaches to recommend videos based on viewers' emotional responses. Their study delves into the incorporation of physical, physiological, and computational analyses to enhance recommendation accuracy.

Some research orient toward computer algorithms and address challenges in efficiently transmitting videos. For example, Wang et al. (2022) focus on factors such as compression algorithms, bandwidth optimization, and latency in the study of vehicle-to-vehicle cooperative video alert dissemination for transmitting accident video in the highway scenario. While their model is primarily on the scenario of vehicle-to-vehicle cooperative video transmitting, their findings offer valuable insights into the technical aspects of video dissemination that may inform broader applications beyond their specific use case.

Our research is the first to specifically focus on online video dissemination from a business perspective. We develop theoretical models to analyze the financial implications of different video content production approaches (in-house production vs. professional outsourcing) and their relevant marketing strategies. The following literature informs and inspires various aspects of our model setup. Li et al. (2019) serve as a primary reference for the development of our study's model. Their work on a two-period decision-making model with a quality formulation similar to ours lays the groundwork for analyzing video dissemination strategies. Liu et al. (2013) provide a robust basis for the demand formulation employed in our study, highlighting key variables and their interdependencies in predicting audience demand. In evaluating the role of promotional efforts in video dissemination, we incorporate the concept from Xie & Wei (2009) and integrate the advertising influence into demand modeling. Additionally, Hartman et al. (2009) provide a framework for considering outsourcing options in video editing, an operational factor that plays a critical role in maintaining quality and scalability in video production. Finally, the expectation equilibrium concept derived from Boianovsky (2022) offers a theoretical lens through which we analyze equilibrium conditions in video dissemination, particularly in terms of balancing content quality with viewer engagement.

### III. MODEL SETUP

#### 3.1. Decision Variables

We consider a content creator publishing videos on a video platform in two periods. In terms of video creation, the content creator has two options: self-editing or outsourcing. We denote the quality of outsourcing video as  $q_o$  and the quality of self-editing video as  $q_s$  in which  $q_o \geq q_s$ , i.e., the quality of the outsourced video is no worse than

that of the self-edited video. However, outsourcing video production to professional editors incurs a cost  $c$  ( $c > 0$ ), while self-editing videos incurs minimal cost and we normalize it to zero.

The content creator also incurs advertising cost for video promotion. The advertising spending amount is denoted as  $a$  in the first period and  $b$  in the second period. In theory, the more a content creator spends on advertising, the more views (extra demand) can be generated. Following the model in Xie & Wei (2009), we use  $P(a) = k\sqrt{a}$  to represent the extra demand generated by advertising in the first period and  $P(b) = k\sqrt{b}$  to represent the extra demand generated by advertising in the second period, where  $k$  is the advertising constant ( $k > 0$ ), reflecting the efficacy of each type of advertising in generating demand.

The decisions the content creator faces are whether to outsource or self-edit videos, and how much to spend on advertising in each period. Model notations can be found in Table 1.

#### 3.2. Demand Formulation

The content creator has  $F$  number of followers. We assume that every follower of the content creator will generate one demand for any published videos. Comparatively, the non-follower number  $\varepsilon$  refers to the number of people on the video platform who are not the content creator's followers and whose demand for the videos is determined by the popularity of video topics and advertising expenses on video promotion. The video demand from non-follower users is denoted as  $\varepsilon t$ , where constant  $t$  measures the topic popularity and  $0 \leq t \leq 1$ . The more popular a topic to viewers, the higher the value of  $t$ , and the higher the video demand from non-followers. Without advertising, the total video demand is solely dependent on the topic of the videos. If the videos address popular or trending topics, they can attract a significant

number of viewers who are not already followers. The base demand  $D_b = F + \varepsilon t$  captures the video demand from both followers and non-followers when there are no advertising promotions. Similar demand formulation can be found in Liu et al. (2014). With the influence of advertising, the first period demand will

increase by  $k\sqrt{a}$ , where  $a$  is the advertising spending amount in the first period (Xie & Wei, 2009). The first period demand  $D_{T_1}$  can be expressed as a summation of base demand and advertising influences:

$$D_{T_1} = D_b + P(a) = F + \varepsilon t + k\sqrt{a}. \quad (1)$$

**TABLE 1. SUMMARY OF MODEL NOTATIONS**

Notation	Definition
$c$	Outsourcing Video Editing Cost, and $c > 0$
$q_a$	Audience Expected Quality in First Period
$q_j$	Video Quality, where $j = s$ or $o$ , $s$ stands for self-editing, $o$ stands for outsourcing
$a$ (decision variable)	First Period Advertising Spending Amount
$b$ (decision variable)	Second Period Advertising Spending Amount
$k$	Advertising Efficacy Constant, $k > 0$
$r$	Reputation Constant in the First Period, $r > 0$
$t$	Topic Relevance Constant, $0 \leq t \leq 1$
$D_b = F + \varepsilon t$	Base Demand (without advertisement influence)
$P(x) = k\sqrt{x}$	Extra Demand Generated by Advertising, where $x = a$ or $b$ , and $a$ is period 1 ad spending, $b$ is period 2 ad spending
$D_{T_i}$	True Demand Formula in Period $i$ , where $i = 1$ or $2$ .
$F$	Content Creator 's Follower Number
$\varepsilon$	Non-follower Number
$m$	Revenue per Demand
$\alpha$	Disappointment Constant on Base Demand
$\beta$	Disappointment Constant on Advertising Efficiency
$\Pi_j$	Total Profit, $s$ stands for self-editing, $o$ stands for outsourcing

We assume the quality reputation of the content creator in the first period  $r$  is a positive constant ( $r > 0$ ). This quality reputation reflects, for instance, historical quality levels of the content creator's videos, and gives the consumer a perception of the quality of the new video introduced in the first period. If the content creator has a high reputation, the corresponding  $r$  value is large, and vice versa. Note that the reputation value  $r$  does not necessarily have a positive relationship with the follower number  $F$ . This means that if a content creator has a high follower number  $F$ , it does not mean that his reputation value is also high. This corresponds to the situation in which a very famous content creator has been exposed in a scandal. Under this scandal scenario, although the follower number  $F$  of that content creator is high, the reputation stays low. The audience's expected quality for the first-period video is influenced by both the creator's reputation and the video promotion effort, and is measured as  $q_a = rP(a) = rk\sqrt{a}$ . Similar formulation can be found in Li et al. (2019).

The demand in the second period is mainly influenced by how well the first period's video quality meets the expectation of audience. Following Li et al. (2019), if the video quality in the first period fails to meet the audiences' expected quality, the disappointment will impact negatively on the video base demand and advertising efficiency in the second period. We denote  $\alpha$  as the audience disappointment constant for base demand, and  $\beta$  as the audience disappointment constant for advertising efficiency. The values of  $\alpha$  and  $\beta$  are between  $[0,1]$ . If the actual quality of the video is higher than the audience's expectation, i.e.,  $q_j - q_a \geq 0$ ,  $j = s$  or  $o$ , then the audience will not be disappointed and the values of  $\alpha$  and  $\beta$  equal to 1, i.e.,  $\alpha = \beta = 1$ . If the actual quality of the video is lower than the audience's expectation, i.e.,  $q_j - q_a \leq 0$ ,  $j = s$  or  $o$ , then the audience will be disappointed with  $0 \leq \alpha < 1$  and  $0 \leq \beta < 1$ . Overall, the values of  $\alpha$  and  $\beta$  can be

either equal to 1 or less than 1 depends on different situations. Incorporating the disappoint effect from audiences' first period experience, the second period demand is expressed as

$$D_{T_2} = \alpha D_b + \beta P(b) = \alpha(F + \varepsilon t) + \beta k\sqrt{b} \quad (2)$$

### 3.3. Profit Formulation

Video content creator's revenue per demand  $m$  represents the amount of money, on average, each unit of demand can generate. We assume the value of  $m$  as a constant, recognizing that its specific value may vary across different platforms depending on their video income algorithms. The revenue for the content creator in the first-period is  $mD_{T_1} = m(F + \varepsilon t) + mk\sqrt{a}$ , and the revenue in the second-period is  $m(\alpha D_b + \beta P(b)) = m\alpha(F + \varepsilon t) + m\beta k\sqrt{b}$ .

The total profit over two periods is the revenue generated across two periods minus all the costs, which include the advertising cost of the first period  $a$ , the advertising cost of the second period  $b$ , and the outsourcing cost  $c$  if the content creator chooses to outsource the videos. The profit for the content creator if the videos are self-edited is

$$\Pi_s = m(F + \varepsilon t) + mk\sqrt{a} + m\alpha(F + \varepsilon t) + m\beta k\sqrt{b} - a - b, \quad (3)$$

and the profit for the outsourcing option is

$$\Pi_o = m(F + \varepsilon t) + mk\sqrt{a} + m\alpha(F + \varepsilon t) + m\beta k\sqrt{b} - a - b - c, \quad (4)$$

where  $a \geq 0$ ,  $b \geq 0$ ,  $c > 0$ ,  $0 \leq \alpha \leq 1$ ,  $0 \leq \beta \leq 1$ .

## IV. MODEL RESULTS

### 4.1. Strategy Decisions of Outsourcing and Self-editing

In this section, we analyze how advertising costs impact video creation strategies. The content creator's decisions on

advertising costs influence both viewer expectations and the realized video quality. We first identify three key ranges of first-period advertising cost ( $a$ ) that lead to different audience responses: (i) With low advertising cost ( $a \leq \frac{q_s^2}{k^2 r^2}$ ), and self-edited video quality meets audience expectations, resulting in no disappointment; (ii) With moderate advertising costs ( $\frac{q_s^2}{k^2 r^2} \leq a \leq \frac{q_o^2}{k^2 r^2}$ ), and self-edited videos disappoint audiences while outsourced videos meet expectations; and (iii) With high advertising costs ( $\frac{q_o^2}{k^2 r^2} \leq a$ ), and both self-edited and outsourced videos fall short of audience expectations, though outsourced videos disappoint less.

Within each advertising cost range, the optimal strategy depends on the dynamics between the first period and the second period decisions. We compare the content creator's profit under each scenario and conclude the following propositions regarding the optimal choice of video production between the self-editing strategy and the outsourcing strategy.

**Proposition 1.**

- (a) *With low advertising cost  $a \leq \frac{q_s^2}{k^2 r^2}$ , regardless of the value of  $b$ , self-editing strategy always generates a higher profit than outsourcing strategy;*
- (b) *With moderate advertising cost  $\frac{q_s^2}{k^2 r^2} \leq a \leq \frac{q_o^2}{k^2 r^2}$ , self-editing strategy generates a higher profit when  $b < \left[ \frac{c-(1-\alpha)m(F+\varepsilon t)}{(1-\beta)mk} \right]^2$ ; otherwise outsourcing strategy generates a higher profit;*
- (c) *With high advertising cost  $\frac{q_o^2}{k^2 r^2} \leq a$ , self-editing strategy generates a higher profit when  $b < \left[ \frac{c-(\alpha_1-\alpha_2)m(F+\varepsilon t)}{(\beta_1-\beta_2)mk} \right]^2$ , otherwise outsourcing strategy generates a higher profit.*

The proof of this proposition and all the following proofs can be found in the Appendix. Proposition 1(a) reveals that with low first-period advertising costs, self-editing is always optimal because both strategies meet audiences' expectations, but self-editing avoids unnecessary costs. Proposition 1(b) shows that with moderate advertising cost in the first period, the second-period advertising cost becomes crucial. A lower second-period cost favors self-editing while a higher cost justifies outsourcing. Proposition 1(c) demonstrates that when both self-editing and outsourcing strategies disappoint audiences, the choice depends on second-period advertising costs, with outsourcing becoming more attractive as costs increase due to its mitigating effect on audience disappointment.

While this section establishes the conditions under which each strategy is superior, the content creator next needs to determine optimal advertising expenditures within these strategic choices, which we discuss in the following subsection.

**4.2. Optimal Advertising Expenditures**

In this subsection, we first obtain the optimal advertising values of  $a$  and  $b$  without any restrictions to serve as theoretical benchmarks. We then derive the optimal advertising expenditures under constraints of advertising budget level, video quality requirements, growth timeline preferences, and resource availability to provide content creators strategic frameworks on advertising investment.

**Theorem 1. (Global Optimization)** *The total profit functions are jointly concave in  $a$  and  $b$ . Hence the global optimal solution  $(a^*, b^*)$  exists with  $a^* = \frac{m^2 k^2}{4}$  and  $b^* = \beta^2 \frac{m^2 k^2}{4}$ .*

While the global optimization provided ideal advertising levels, the content creator must make decisions within practical constraints. These constraints arise from how advertising

levels affect audience expectations and potential disappointment. We first define the following threshold values that help determine optimal strategies in Table 2. We next analyze

five key scenarios that the content creator faces. Table 3 summarizes the five scenarios and their corresponding Theorem.

**TABLE 2. SUMMARY OF CONSTRAINT SCENARIOS**

Quality-advertising balance threshold for self-editing	$\lambda_1 = 1 - \frac{1}{2}mk \frac{kr}{q_s}$
Quality-advertising balance threshold for outsourcing	$\lambda_2 = \frac{1}{2}mk \frac{kr}{q_o} - 1$
Advertising efficiency threshold	$\lambda_3 = \frac{1}{2}mk \frac{1}{\delta} - 1$
Adjusted efficiency threshold with disappointment	$\lambda_4 = \frac{1}{2}m\beta k \frac{1}{\delta} - 1$

**TABLE 3. SUMMARY OF CONSTRAINT SCENARIOS**

Scenario Condition	Scenario Description	Corresponding Theorem
$a \leq \frac{q_s^2}{k^2r^2}$	self-editing when first-period advertising cost is low	Theorem 2
$\frac{q_s^2}{k^2r^2} \leq a \leq \frac{q_o^2}{k^2r^2} \ \& \ b \leq \delta^2$	self-editing when first-period advertising cost is moderate and second-period cost is low	Theorem 3
$\frac{q_s^2}{k^2r^2} \leq a \leq \frac{q_o^2}{k^2r^2} \ \& \ b \geq \delta^2$	outsourcing when first-period advertising cost is moderate and second-period cost is high	Theorem 4
$\frac{q_o^2}{k^2r^2} \leq a \ \& \ b \leq \delta^{*2}$	self-editing, when first-period advertising cost is high and second-period advertising cost is low	Theorem 5
$\frac{q_o^2}{k^2r^2} \leq a \ \& \ b \geq \delta^{*2}$	outsourcing when first-period advertising cost is high and second-period cost is high	Theorem 6

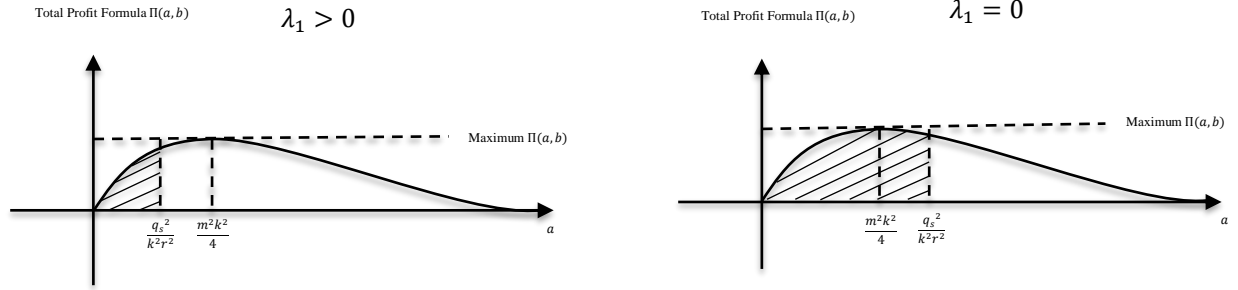
**Theorem 2.** When  $a \leq \frac{q_s^2}{k^2r^2}$ , for the first-period advertising spending,

- i) If  $\lambda_1 > 0$ , the constrained optimal value is  $a^* = \frac{q_s^2}{k^2r^2}$ ;
- ii) If  $\lambda_1 = 0$ , the constrained optimal value is  $a^* = \frac{m^2k^2}{4}$ ;

And the second-period advertising spending is  $b^* = \frac{m^2k^2}{4}$ .

The results of Theorem 2 are illustrated in Graph 1 below. Note that the shaded area in Graph 1 and all the following graphs refers to the feasible ranges of  $a$  or  $b$  in the corresponding Theorem.





**GRAPH 1. RANGE OF  $a$**

Note that  $\lambda_1 > 0$  is equivalent to  $q_s > mrk^2/2$ . Graph 1 shows that if the self-editing video quality is higher than a threshold value, the content creator can spend less than the global optimal value in advertising in the first period to maximize his profit. This finding suggests that content creators with quality self-editing capabilities should moderate their first-period advertising to avoid raising audiences' quality expectations beyond the actual quality level.

**Theorem 3.** When  $\frac{q_s^2}{k^2r^2} \leq a \leq \frac{q_o^2}{k^2r^2}$ ,  $b \leq \delta^2$  where  $\delta = \frac{c-(1-\alpha)m(F+\epsilon t)}{(1-\beta)mk}$ ,

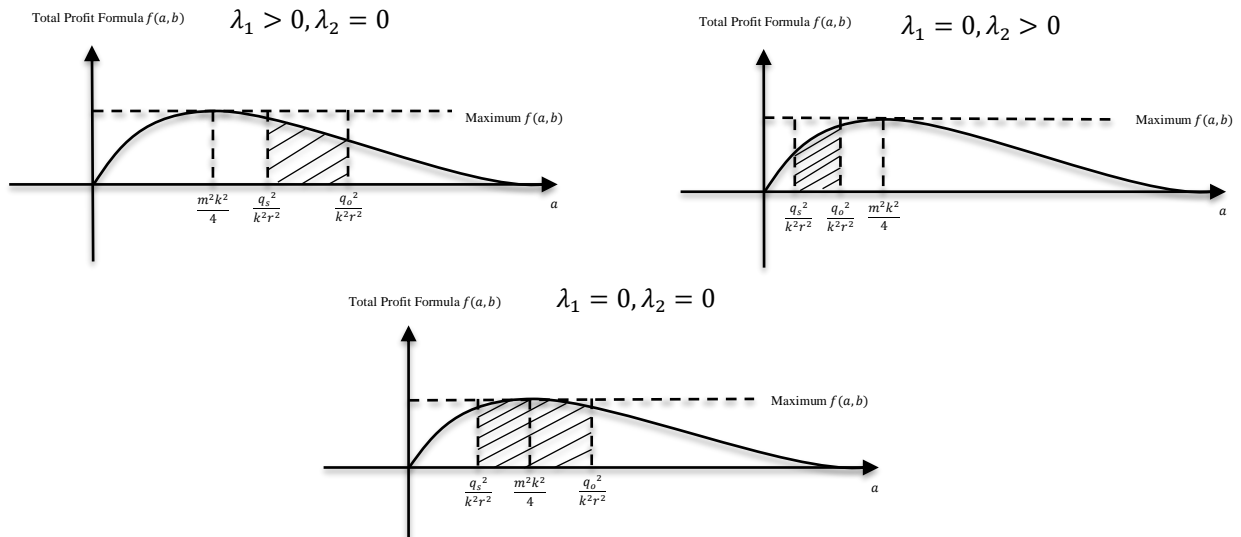
For the first-period advertising spending:

- i) If  $\lambda_1 > 0$  and  $\lambda_2 = 0$ , the constrained optimal value is  $a^* = \frac{q_s^2}{k^2r^2}$ ;
- ii) If  $\lambda_1 = 0$  and  $\lambda_2 > 0$ , the constrained optimal value is  $a^* = \frac{q_o^2}{k^2r^2}$ ;
- iii) If  $\lambda_1 = 0$  and  $\lambda_2 = 0$ , the constrained optimal value is  $a^* = \frac{m^2k^2}{4}$ ;

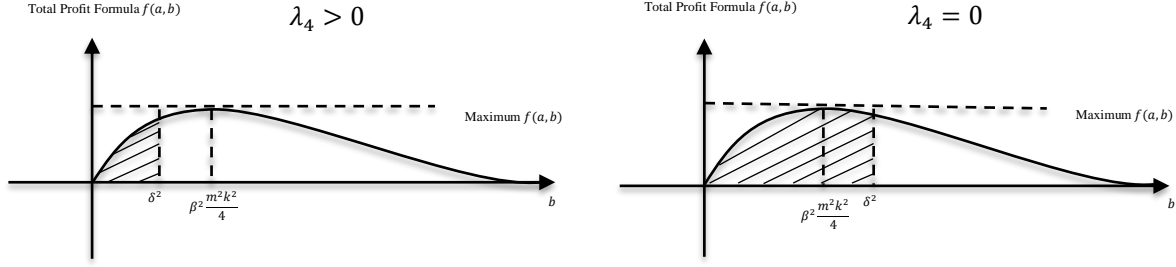
For the second-period advertising spending:

- iv) If  $\lambda_4 > 0$ , the constrained optimal value is  $b^* = \delta^2$ ;
- v) If  $\lambda_4 = 0$ , the constrained optimal value is  $b^* = \beta^2 \frac{m^2k^2}{4}$ .

The results of Theorem 3 are illustrated in Graph 2A and Graph 2B below.



**GRAPH 2A. RANGE OF  $a$**



**GRAPH 2B. RANGE OF  $b$**

In Graph 2A, note that  $\lambda_1 > 0$  is equivalent to  $q_s > mrk^2/2$  and  $\lambda_2 > 0$  is equivalent to  $q_o < mrk^2/2$ . Both the scenarios  $\lambda_1 > 0, \lambda_2 = 0$  and  $\lambda_1 = 0, \lambda_2 > 0$  represent the situation that the self-editing video quality is worse than that from outsourcing. Under this situation, the profit is less than the global optimal value. The scenario  $\lambda_1 = 0$  and  $\lambda_2 = 0$  shows that video quality from either strategy is the same and the content creator's profit reaches the global optimal value. In Graph 2B, note that  $\lambda_4 > 0$  is equivalent to  $k > 2\delta/(m\beta)$ , indicating that when advertising efficacy  $k$  is greater than a certain threshold value, the content creator can save on advertising cost in the second period.

These findings demonstrate how the relative quality differences between production strategies directly influence optimal advertising decisions: when quality levels are balanced, content creators can advertise at optimal levels, but when quality differences exist, they must adjust their advertising to boundary levels to maximize profit.

**Theorem 4.** When  $\frac{q_s^2}{k^2 r^2} \leq a \leq \frac{q_o^2}{k^2 r^2}$ ,  $b \geq \delta^2$  where  $\delta = \frac{c-(1-\alpha)m(F+\epsilon t)}{(1-\beta)mk}$ ,

For the first-period advertising spending:

- i) If  $\lambda_1 > 0$  and  $\lambda_2 = 0$ , the constrained optimal value is  $a^* = \frac{q_s^2}{k^2 r^2}$ ;
- ii) If  $\lambda_1 = 0$  and  $\lambda_2 > 0$ , the constrained optimal value is  $a^* = \frac{q_o^2}{k^2 r^2}$ ;

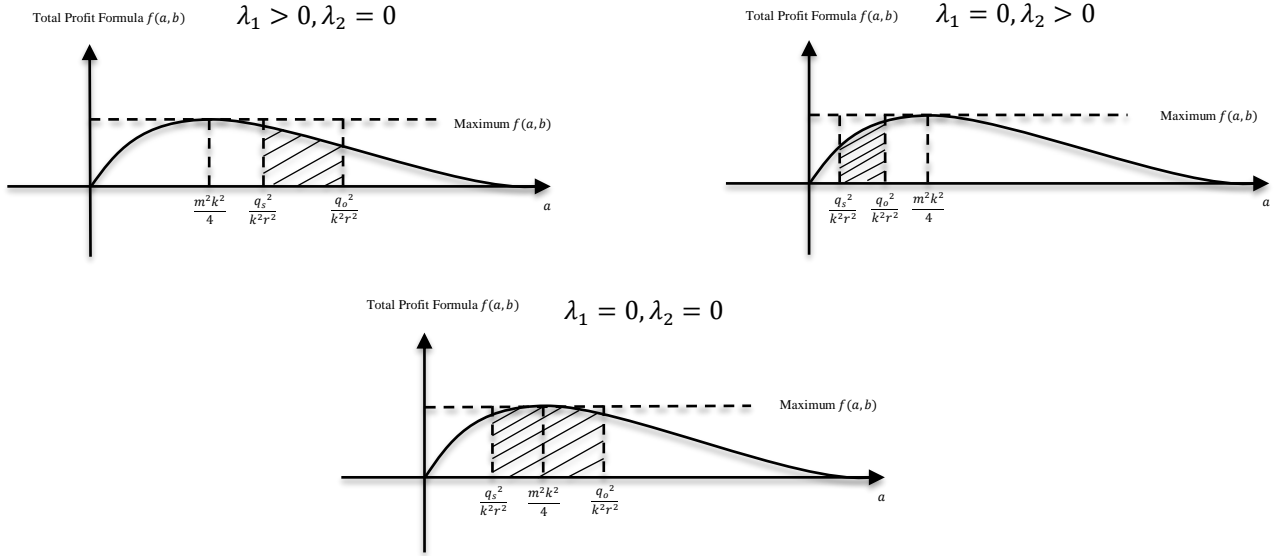
iii) If  $\lambda_1 = 0$  and  $\lambda_2 = 0$ , the constrained optimal value is  $a^* = \frac{m^2 k^2}{4}$ ;

For the second-period advertising spending:

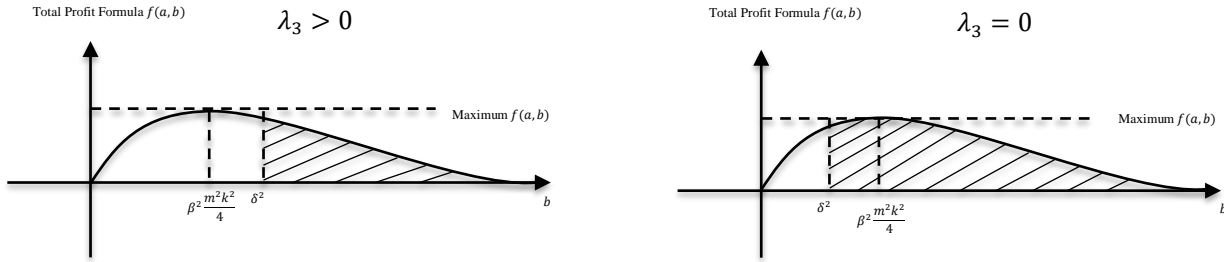
- iv) If  $\lambda_3 > 0$ , the constrained optimal value is  $b^* = \delta^2$ ;
- v) If  $\lambda_3 = 0$ , the constrained optimal value is  $b^* = \frac{m^2 k^2}{4}$ .

The results of Theorem 4 are illustrated in Graph 3A and Graph 3B below.

Theorem 4 has the same restriction on the first-period advertising spending as in Theorem 3, but requires a higher advertising efficacy in the second-period (note that  $\lambda_3 > 0$  is equivalent to  $k > 2\delta/m$ , which is greater than the efficacy threshold in Theorem 3). This finding reveals important strategic considerations for content creators considering the strategy of outsourcing. Despite the higher video quality from outsourcing, content creators must still exercise caution with their first-period advertising spending in order to manage audience expectations effectively. The quality premium from outsourcing offers greater strategic flexibility in the second-period advertising spending. Content creators can be more aggressive in their advertising approach when their content quality provides a buffer against audience disappointment.



**GRAPH 3A. RANGE OF  $a$**



**GRAPH 3B. RANGE OF  $b$**

**Theorem 5.** When  $\frac{q_0^2}{k^2 r^2} \leq a, b \leq \delta^{*2}$  where  $\delta^* = \frac{c - (\alpha_1 - \alpha_2)m(F + \epsilon t)}{(\beta_1 - \beta_2)mk}$ ,

For the first-period advertising spending:

i) If  $\lambda_1 > 0$ , the constrained optimal value is

$$a^* = \frac{q_0^2}{k^2 r^2};$$

ii) If  $\lambda_1 = 0$ , the constrained optimal value is

$$a^* = \frac{m^2 k^2}{4};$$

For the second-period advertising spending:

iii) If  $\lambda_3 > 0$ , the constrained optimal value is

$$b^* = \delta^{*2};$$

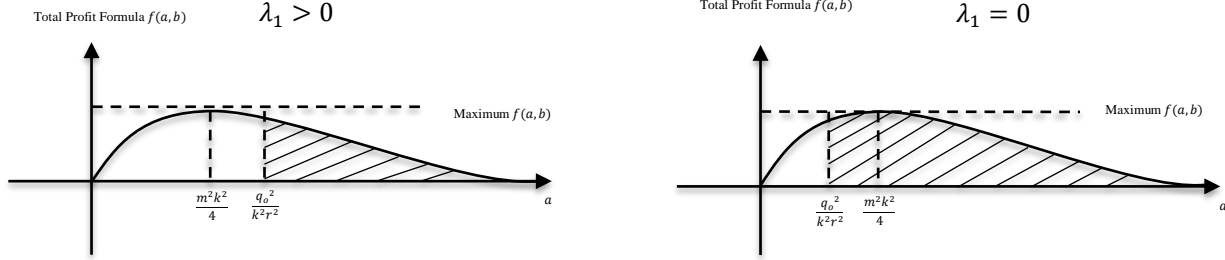
iv) If  $\lambda_3 = 0$ , the constrained optimal value is

$$b^* = \beta_2^2 \frac{m^2 k^2}{4}.$$

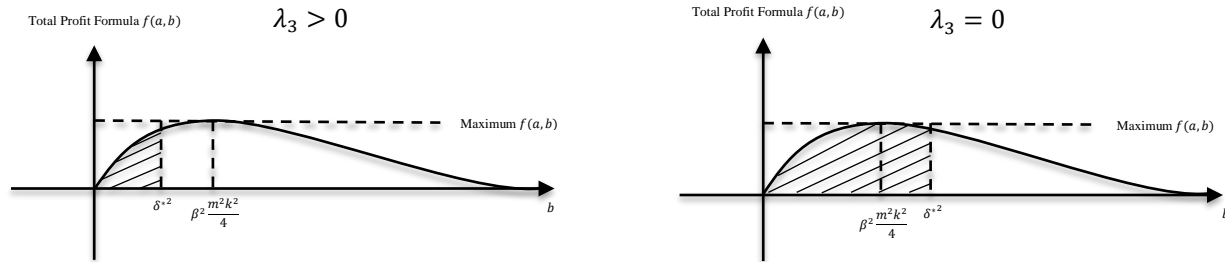
The results of Theorem 5 are illustrated in Graph 4A and Graph 4B below.

Theorem 5 examines the scenario where the first-period advertising spending is high ( $\frac{q_0^2}{k^2 r^2} \leq a$ ) and the second-period advertising spending is below the disappointment threshold ( $b \leq \delta^{*2}$ ). The results imply that, in the first-period, when  $\lambda_1 > 0$ , content creators should advertise at the lower boundary cost ( $a = \frac{q_0^2}{k^2 r^2}$ ), but when  $\lambda_1 = 0$ , they should spend on advertising at the global optimal level ( $a = \frac{m^2 k^2}{4}$ ). For the second-period, advertising expenditure should be set at the threshold level

( $b = \delta^{*2}$ ) when  $\lambda_3 > 0$ , and at the adjusted optimal level ( $b = \beta_2^2 \frac{m^2 k^2}{4}$ ) when  $\lambda_3 = 0$ .



**GRAPH 4A. RANGE OF  $a$**



**GRAPH 4B. RANGE OF  $b$**

This finding reveals several important strategic implications. When content creators choose self-editing in high advertising cost scenarios, they must be particularly vigilant about managing audience expectations. The lower quality associated with self-editing (reflected in  $\beta_2$ ) requires more conservative advertising spending in the second period to prevent audience disappointment. Unlike outsourcing, which provides a quality buffer, self-editing at high advertising cost levels requires careful balancing of advertising intensity against quality capabilities to maintain audience satisfaction.

**Theorem 6.** When  $\frac{q_0^2}{k^2 r^2} \leq a$ ,  $b \geq \delta^{*2}$  where  $\delta^* = \frac{c - (\alpha_1 - \alpha_2)m(F + \epsilon t)}{(\beta_1 - \beta_2)mk}$   
For the first-period advertising spending:

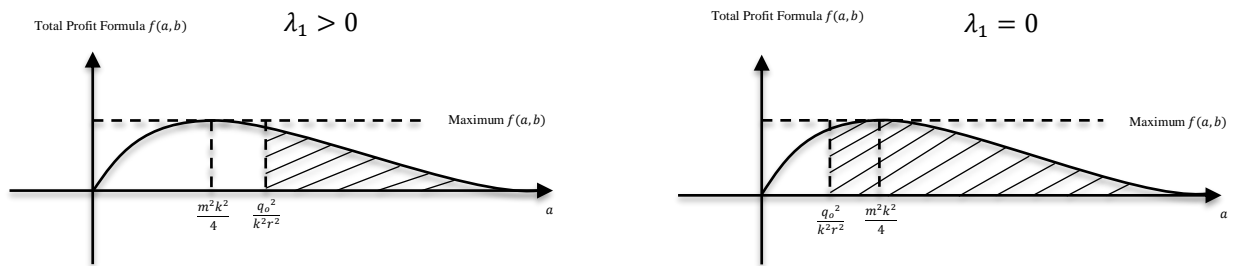
- i) If  $\lambda_1 > 0$ , the constrained optimal value is  $a^* = \frac{q_0^2}{k^2 r^2}$ ;
  - ii) If  $\lambda_1 = 0$ , the constrained optimal value is  $a^* = \frac{m^2 k^2}{4}$ ;
- For the second-period advertising spending:
- iii) If  $\lambda_2 > 0$ , the constrained optimal value is  $b^* = \delta^{*2}$ ;
  - iv) If  $\lambda_2 = 0$ , the constrained optimal value is  $b^* = \beta_1^2 \frac{m^2 k^2}{4}$ .

The results of Theorem 6 are illustrated in Graph 5A and Graph 5B below.

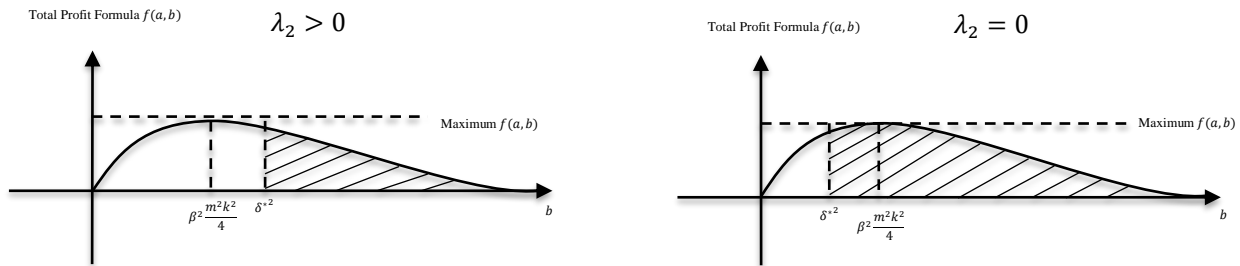
Theorem 6 analyzes the scenario where the first-period advertising is high ( $\frac{q_0^2}{k^2 r^2} \leq a$ ) and the second-period advertising cost exceeds the disappointment threshold ( $b \geq \delta^{*2}$ ). The theorem provides optimal advertising recommendations based on quality and

effectiveness parameters. For the first-period, when video quality significantly influences advertising effectiveness, i.e.,  $\lambda_1 > 0$ , content creators should advertise at the lower boundary cost of the high range  $a = \frac{q_0^2}{k^2\gamma^2}$ . However, when quality and advertising cost are well-balanced at  $\lambda_1 = 0$ , they can have advertising spending at the global optimal level  $a = \frac{m^2k^2}{4}$ . For the second-period, when advertising effectiveness is high relative to quality differences, i.e.,  $\lambda_2 >$

0, content creators should maintain advertising spending at the threshold level  $b = \delta^{*2}$ . When advertising and quality effects are balanced at  $\lambda_2 = 0$ , they can advertise at an adjusted optimal cost level  $b = \beta_1^2 \frac{m^2k^2}{4}$ . These findings highlight that in high-advertising cost scenarios, outsourcing provides important protection against audience disappointment through its quality buffer, enabling more aggressive second-period advertising spending compared to self-editing strategies.



**GRAPH 5A. RANGE OF  $a$**



**GRAPH 5B. RANGE OF  $b$**

### 4.3. Decision Framework

Our theoretical analysis above reveals systematic patterns in how content creators should adjust their advertising strategies based on multiple factors. These include production quality levels, advertising costs in both periods, reputation effects, audience disappointment levels, and advertising effectiveness. Graph 6 provides a comprehensive decision framework that synthesizes all results, guiding content creators through optimal strategy choices based

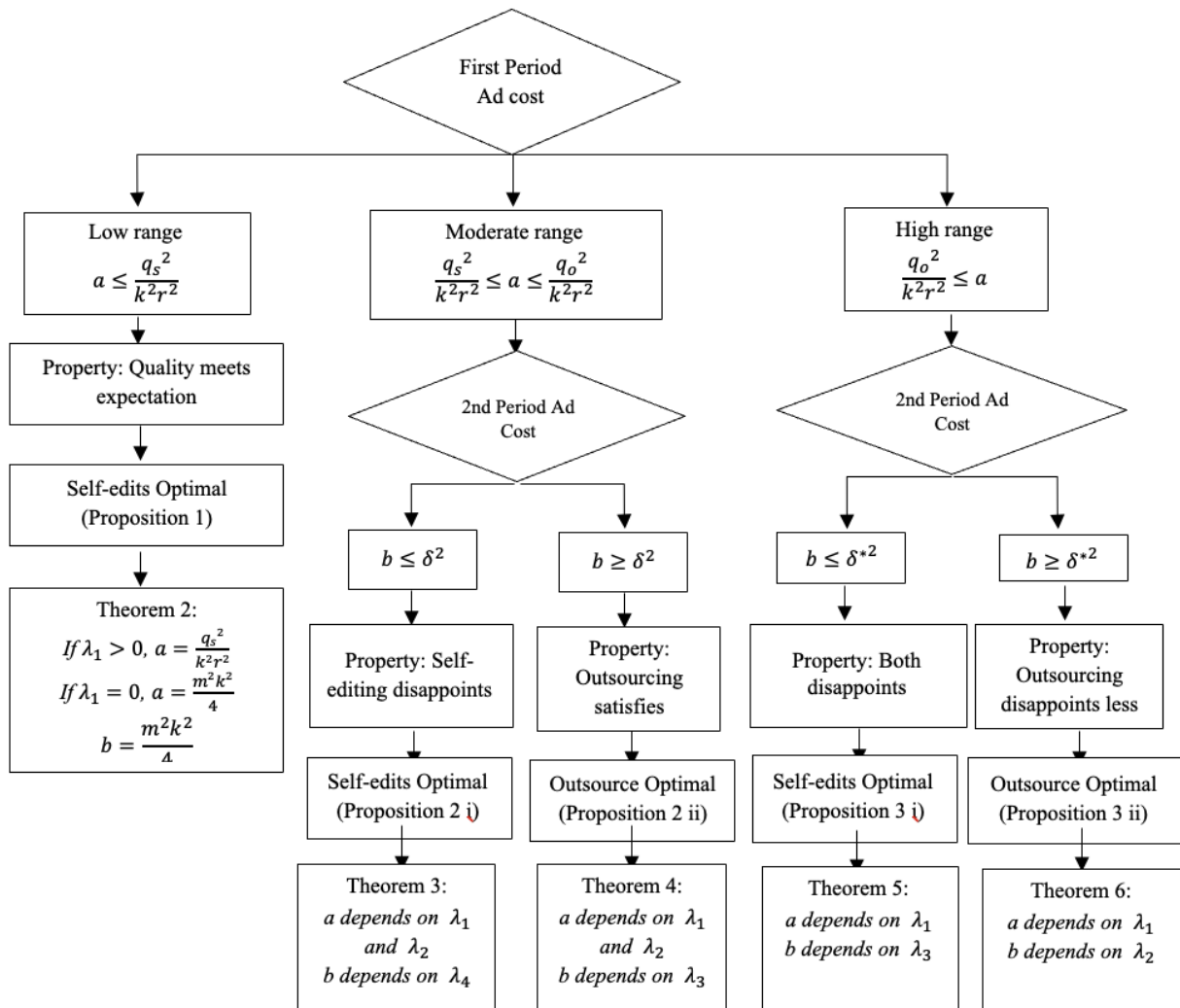
on their specific circumstances. This framework integrates the theoretical findings about advertising thresholds, quality requirements, and audience expectations into practical decision paths.

This decision framework provides significant managerial insights. Specifically, content creators should be mindful that investing in higher production quality may require balancing with advertising costs to avoid overburdening their budget. The reputation effects are critical, as past content quality can influence audience expectations and

response to new content. This requires creators to align their marketing efforts with the current level of audience satisfaction. Creators should also recognize that audience disappointment can lead to diminishing returns on advertising efforts, and it is important to effectively manage audience expectations through transparent communication of video quality.

This framework has far-reaching implications not only for individual creators but also for broader industry dynamics. By adopting these strategic insights, content creators can optimize their advertising efforts, leading to

sustainable growth and stronger viewer loyalty. Additionally, advertisers can leverage these insights to better understand the relationship between content quality and audience engagement, ultimately refining their marketing tactics across various platforms. This framework may also influence advertising standards and expectations in digital content ecosystems, and possibly impact the future of content creation.



GRAPH 6. DECISION FRAMEWORK

## V. CONCLUSION

The online video industry has experienced unprecedented growth, particularly during the COVID-19 pandemic, establishing content creation as a viable career path. This paper develops a two-period model examining content creators' strategic decisions regarding video production and advertising. Our analysis yields several important theoretical and practical insights for content creators.

First, our analysis reveals how advertising costs fundamentally shape the optimal choice between self-editing and outsourcing videos. When advertising costs are low, self-editing emerges as the consistently optimal strategy since both approaches can meet audience expectations. As advertising levels increase to moderate levels, the choice becomes more complex and strategic, depending critically on planned second-period advertising intensity. At high advertising levels, outsourcing becomes increasingly valuable as a buffer against audience disappointment, particularly when aggressive advertising is planned.

Second, our findings provide a comprehensive decision framework that guides content creators through their strategic choices. We first establish optimal advertising expenditure levels that serve as theoretical benchmarks without constraints. Then with considerations of advertising budget levels, quality requirements determined by advertising intensity, growth timeline preferences, and resource availability, the framework recommends different strategic paths based on advertising investment levels.

These findings provide both theoretical understanding and practical guidance for content creators navigating the growing online video industry. Our decision framework offers clear strategic paths based on creators' resources, ambitions, and market conditions, helping them make informed choices in this dynamic digital landscape.

**Disclaimer:** The views expressed in this article are those of the authors and do not reflect the official policy or position of the US Air Force, Department of Defense, or US Government.

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## APPENDIX

### Proof of Proposition 1.

(a) When  $a \leq \frac{q_s^2}{k^2 r^2}$ , the video quality in the first period does not disappoint audiences in both outsourcing and self-editing strategies. Thus,  $\alpha = \beta = 1$  in both strategies. The total profit of the self-editing strategy would be  $m(F + \epsilon t) + mk\sqrt{a} + m(F + \epsilon t) + mk\sqrt{b} - a - b$ , where  $a \geq 0$ ,  $b \geq 0$ . The total profit of the outsourcing strategy would be  $m(F + \epsilon t) + mk\sqrt{a} + m(F + \epsilon t) + mk\sqrt{b} - a - b - c$ , where  $a \geq 0$ ,  $b \geq 0$ ,  $c > 0$ . Since the total profit formula of the outsourcing strategy has one more cost  $c$ , the self-editing strategy will always have a higher profit in this range regardless of the value  $b$ .



(b) When  $\frac{q_s^2}{k^2r^2} \leq a \leq \frac{q_o^2}{k^2r^2}$ , the self-editing video quality disappoints audiences. Thus,  $0 \leq \alpha < 1$  and  $0 \leq \beta < 1$  in the self-editing strategy. However, if the video is outsourced, the video quality does not disappoint audiences. Thus,  $\alpha = \beta = 1$  in the outsourcing strategy. The total profit formula of the self-editing strategy would be  $m(F + \varepsilon t) + mk\sqrt{a} + m\alpha(F + \varepsilon t) + m\beta k\sqrt{b} - a - b$ , where  $a \geq 0$ ,  $b \geq 0$ ,  $0 \leq \alpha < 1$ ,  $0 \leq \beta < 1$ . The total profit formula of the outsourcing strategy would be  $m(F + \varepsilon t) + mk\sqrt{a} + m(F + \varepsilon t) + mk\sqrt{b} - a - b - c$ , where  $a \geq 0$ ,  $b \geq 0$ ,  $c > 0$ . If the self-editing strategy has higher profit than outsourcing, then we have  $m(1 + \alpha)(F + \varepsilon t) + mk\sqrt{a} + m\beta k\sqrt{b} - a - b > 2m(F + \varepsilon t) + mk\sqrt{a} + mk\sqrt{b} - a - b - c$ . Simplify this inequality, we get  $(1 - \alpha)m(F + \varepsilon t) + (1 - \beta)mk\sqrt{b} < c$ . Note that  $a$  is canceled out in the inequality. Solving for  $b$  we get  $b < \left(\frac{c - (1 - \alpha)m(F + \varepsilon t)}{(1 - \beta)mk}\right)^2$ . Therefore, when  $\frac{q_s^2}{k^2r^2} \leq a \leq \frac{q_o^2}{k^2r^2}$ , and  $b < \left(\frac{c - (1 - \alpha)m(F + \varepsilon t)}{(1 - \beta)mk}\right)^2$ , the self-editing strategy generates higher profit. Similarly, if the outsourcing strategy generates higher profit. Then we have  $m(1 + \alpha)(F + \varepsilon t) + mk\sqrt{a} + m\beta k\sqrt{b} - a - b < 2m(F + \varepsilon t) + mk\sqrt{a} + mk\sqrt{b} - a - b - c$ . Solving for  $b$  we get  $b > \left(\frac{c - (1 - \alpha)m(F + \varepsilon t)}{(1 - \beta)mk}\right)^2$ .

(c) When  $\frac{q_o^2}{k^2r^2} \leq a$ , both self-editing and outsourcing quality will disappoint audiences. However, since outsourcing video has a higher quality, the values of  $\alpha$  and  $\beta$  in outsourcing strategy are also higher. Suppose  $\alpha_1$  and  $\beta_1$  are the audience disappointment constants in the outsourcing strategy;  $\alpha_2$  and  $\beta_2$  are the audience disappointment constants of the self-editing strategy. Thus, we have  $0 \leq \alpha_2 < \alpha_1 < 1$  and  $0 \leq \beta_2 < \beta_1 < 1$ . The total profit formula of the self-editing strategy would be  $m(F + \varepsilon t) + mk\sqrt{a} + m\alpha_2(F + \varepsilon t) + m\beta_2 k\sqrt{b} - a - b$ , where  $a \geq 0$ ,  $b \geq 0$ ,  $0 \leq$

$\alpha_2 < \alpha_1 < 1$ ,  $0 \leq \beta_2 < \beta_1 < 1$ . The total profit formula of the outsourcing strategy would be  $m(F + \varepsilon t) + mk\sqrt{a} + m\alpha_1(F + \varepsilon t) + m\beta_1 k\sqrt{b} - a - b - c$ , where  $a \geq 0$ ,  $b \geq 0$ ,  $c > 0$ ,  $0 \leq \alpha_2 < \alpha_1 < 1$ ,  $0 \leq \beta_2 < \beta_1 < 1$ . If self-editing has higher profit than outsourcing; then we have  $m(1 + \alpha_2)(F + \varepsilon t) + mk\sqrt{a} + m\beta_2 k\sqrt{b} - a - b > m(1 + \alpha_1)(F + \varepsilon t) + mk\sqrt{a} + m\beta_1 k\sqrt{b} - a - b - c$ . Simplify this inequality, we get  $(\alpha_1 - \alpha_2)m(F + \varepsilon t) + (\beta_1 - \beta_2)mk\sqrt{b} < c$ . Note that  $a$  is canceled out in the inequality. Solving for  $b$  we get  $b < \left(\frac{c - (\alpha_1 - \alpha_2)m(F + \varepsilon t)}{(\beta_1 - \beta_2)mk}\right)^2$ . Therefore, when  $\frac{q_o^2}{k^2r^2} \leq a$ ,  $b < \left(\frac{c - (\alpha_1 - \alpha_2)m(F + \varepsilon t)}{(\beta_1 - \beta_2)mk}\right)^2$ , the self-editing strategy generates higher profit. Similarly, if the outsourcing strategy generates higher profit. Then  $m(1 + \alpha_2)(F + \varepsilon t) + mk\sqrt{a} + m\beta_2 k\sqrt{b} - a - b < m(1 + \alpha_1)(F + \varepsilon t) + mk\sqrt{a} + m\beta_1 k\sqrt{b} - a - b - c$ . Solving for  $b$ , we get  $b > \left(\frac{c - (\alpha_1 - \alpha_2)m(F + \varepsilon t)}{(\beta_1 - \beta_2)mk}\right)^2$ .

### Proof of Theorem 1.

We have  $\Pi(a, b) = m(F + \varepsilon t) + mk\sqrt{a} + m\alpha(F + \varepsilon t) + m\beta k\sqrt{b} - a - b - c$ . We first prove that this function is concave down so that the solution to First Order Condition provides the global optimum.

For  $a$  we have:  $\frac{\partial}{\partial a} \Pi(a, b) = \frac{1}{2} mka^{-\frac{1}{2}} - 1$  and  $\frac{\partial^2}{\partial a^2} \Pi(a, b) = -\frac{1}{4} mka^{-\frac{3}{2}}$ . Since  $m > 0, k > 0$ , then  $-\frac{1}{4} mka^{-\frac{3}{2}} < 0$  for  $a > 0$ . Therefore, the second partial derivative of  $f(a, b)$  respect to  $a$  is smaller than 0 for all  $a > 0$ . Thus, the function respect to  $a$  is concave down as  $a > 0$ . We let  $\frac{\partial}{\partial a} \Pi(a, b) = \frac{1}{2} mka^{-\frac{1}{2}} - 1 = 0$ . Solve it and we get  $a = \frac{m^2 k^2}{4}$ .

For  $b$ , we have similar processes:  $\frac{\partial}{\partial b} \Pi(a, b) = \frac{1}{2} m\beta k b^{-\frac{1}{2}} - 1$  and  $\frac{\partial^2}{\partial b^2} \Pi(a, b) =$

$-\frac{1}{4}m\beta kb^{-\frac{3}{2}}$ . Since  $m > 0, k > 0, \beta > 0$ , then  $-\frac{1}{4}m\beta kb^{-\frac{3}{2}} < 0$  for  $b > 0$ . Therefore, the second partial derivative of  $\Pi(a, b)$  respect to  $b$  is smaller than 0 for all  $b > 0$ . Thus, the function respect to  $b$  is concave down as  $b > 0$ . We let  $\frac{\partial}{\partial b}\Pi(a, b) = \frac{1}{2}m\beta kb^{-\frac{1}{2}} - 1 = 0$ . Solve it and we get  $b = \beta^2 \frac{m^2 k^2}{4}$ .

**Proof of Theorem 2.**

We involve the Karush–Kuhn–Tucker (KKT) method to calculate the optimal values of  $a$  and  $b$ . The profit formula and conditions:

$$\begin{aligned} &\Pi(a, b) \\ &= 2m(F + \varepsilon t) + mk\sqrt{a} + m\alpha(F + \varepsilon t) \\ &+ m\beta k\sqrt{b} - a - b \\ &a \leq \frac{q_s^2}{k^2 r^2} \text{ convert into } \frac{q_s^2}{k^2 r^2} - a \geq 0 \text{ and } a \geq 0, \\ &b \geq 0. \end{aligned}$$

Thus, construct the Lagrange equation and conditions:

$$\begin{aligned} &l(a, b, \lambda) \\ &= 2m(F + \varepsilon t) + mk\sqrt{a} + m\alpha(F + \varepsilon t) \\ &+ m\beta k\sqrt{b} - a - b + \lambda_1\left(\frac{q_s^2}{k^2 r^2} - a\right) + \lambda_2 a \\ &+ \lambda_3 b \\ &\frac{\partial}{\partial a} l(a, b, \lambda) = \frac{1}{2}mk \frac{1}{\sqrt{a}} - 1 - \lambda_1 + \lambda_2 = 0 \\ &\text{(condition 1)} \\ &\frac{\partial}{\partial b} l(a, b, \lambda) = \frac{1}{2}mk \frac{1}{\sqrt{b}} - 1 + \lambda_3 = 0 \\ &\text{(condition 2)} \end{aligned}$$

$$\begin{aligned} &\lambda_1\left(\frac{q_s^2}{k^2 r^2} - a\right) = 0 \text{ (condition 3)} \\ &\lambda_2 a = 0 \text{ (condition 4)} \\ &\lambda_3 b = 0 \text{ (condition 5)} \end{aligned}$$

Since  $a$  and  $b$  in condition 1 and condition 2 are on the denominator, thus we can get additional condition 6:  $a > 0, b > 0, \lambda_1 \geq 0, \lambda_2 = 0, \lambda_3 = 0$  (condition 6)

Thus, we have two situations here:

Situation 1:  $\lambda_1 > 0, \lambda_2 = 0, \lambda_3 = 0$

$\frac{q_s^2}{k^2 r^2} - a = 0$  from condition 3 and we have  $a = \frac{q_s^2}{k^2 r^2}$ ;  $\frac{1}{2}mk \frac{1}{\sqrt{b}} - 1 = 0$  from condition 2 and we obtain  $b = \frac{m^2 k^2}{4}$ . In addition,  $\frac{1}{2}mk \frac{1}{\sqrt{a}} - 1 - \lambda_1 = 0$  from condition 1 with  $a = \frac{q_s^2}{k^2 r^2}$ , and we have  $\lambda_1 = \frac{1}{2}mk \frac{kr}{q_s} - 1$ . Since  $\lambda_1 > 0$ , thus we have  $\frac{1}{2}mk \frac{kr}{q_s} - 1 > 0$ . Simplify it we have  $\frac{k^2 r^2}{q_s^2} > \frac{4}{m^2 k^2}$ . Thus,  $\frac{q_s^2}{k^2 r^2} < \frac{m^2 k^2}{4}$ .

Situation 2:  $\lambda_1 = 0, \lambda_2 = 0, \lambda_3 = 0$

$\frac{1}{2}mk \frac{1}{\sqrt{a}} - 1 = 0$  from condition 1, and we have  $a = \frac{m^2 k^2}{4}$ ;  $\frac{1}{2}mk \frac{1}{\sqrt{b}} - 1 = 0$  from condition 2 and we have  $b = \frac{m^2 k^2}{4}$ .

The proofs of Theorems 3, 4, 5, and 6 are similar to that of Theorem 2 using the Karush–Kuhn–Tucker method. We omit here.