Analysis of Vehicle Routing Problem with Stochastic Travel and Service Times: A Case Study

Abdel-Aziz M. Mohamed*
Arab Academy for Science, Technology and Maritime Transport, Cairo, Egypt

Transportation is an essential driver of the supply chain; directly impacts both the responsiveness and efficiency therein. The decision of route selection based on the optimal travel time reduces the product delivery time and cost; enhancing customer satisfaction. This paper presents a case study identifying the optimal routes in a network consisting of 81 alternatives. Various sample data were collected in regards to travel time between adjacent nodes as well as the loading and unloading times for each node. A probability distribution function was identified using a curve fitting technique for each sample. Pro-Model simulation software was executed to determine the average travel time and standard deviation for all possible routes. The sensitivity analysis outcome illustrates that as the standard deviation of the travel time between adjacent nodes increases; the average travel time of the entire trip increases, standard deviation had no specific trend, and optimal route was subject to change.
Keywords: supply chain, vehicle routing, simulation modeling, statistical analysis

* Corresponding Author. E-mail address: amohamed55@yahoo.com

I. INTRODUCTION

Vehicle routing and scheduling decisions are particularly crucial transportation-related decisions taken in the supply chain as they result in a decrease of transportation and delivery costs whilst providing equal levels of responsiveness as per customer request (Chopra and Meindl, 2004, page 436). If these decisions are not carefully made, they could result in exceedingly high costs, prolonged travel time and minimal customer satisfaction. In this case study, the routing and scheduling decisions are based on travel time. The selected routes leading to the shortest travel time improve the responsiveness of the supply chain. Therefore if those decisions are carefully made, they will in turn improve the responsiveness and efficiency of the supply chain simultaneously.

Due to complications of modern transportation networks and the stochastic nature of travel and service times (loading and unloading times); determining an exact solution to the vehicle routing problem is difficult. Accordingly, this research considers using the Pro-Model simulation software (Harrell, Ghosh, and Bowden, 2000) to predict the behavior of the transportation system and select the optimal routes that minimize the total time required to visit all nodes including loading and unloading times.

This paper presents a case study to determine the optimal route of the transportation network used by the vehicles of a multi-national organization operating in Cairo, Egypt. Due to the student version limitations of the Pro-model software, the network considered in this study will consist of 5 nodes; a source (warehouse), 3 transshipments, and a destination. There are 3 possible routes between every 2 adjacent nodes, 81 routes were considered in this study. Company vehicles load merchandise from the warehouse then deliver them to the other nodes in the network. The travel times for all routes and the loading and unloading times for nodes each of which following a probability distribution function. The objective is to select the routes that minimize the total travel times; from the moment the vehicle arrives at the main warehouse until it completes service at the destination node.

Fig. 1 shows the locations of the main warehouse and nodes (branches) to be visited by the company vehicle. The stochastic assumption of the vehicle inter-arrival times (to the warehouse), travel times (between adjacent branches) and loading and unloading times (at each branch) were considered. This research illustrates the process of building a simulation model of which determines the optimal solutions to the routing problem via the Pro-Model software. The simulation software will generate a sample of travel times for each route. ANOVA will then determine the routes with minimal travel time, lacking significant differences. Finally, post-optimality analysis of the results will be presented.

II. LITERATURE REVIEW

Vehicle routing and scheduling decisions are critical in supply chain management due to the fact that they directly impact the efficiency and responsiveness of the supply chain. There is a quantum of research covering vehicle routing problem recently. This section will review some of that work.

Carraway et al. (2001) have studied a special case of the stochastic traveling salesman problem with independent and normally distributed travel times. They considered the maximization of the probability of completing the tour by the due date. Cordeau, Laporte,and Mercier (2001) have presented a unified tabu search heuristic for the vehicle routing problems with time windows. Cordeau, Gendreau, Laporte, Potvin, and Semet (2002) presented a survey on the heuristic algorithms developed to provide solution to the vehicle routing problems.

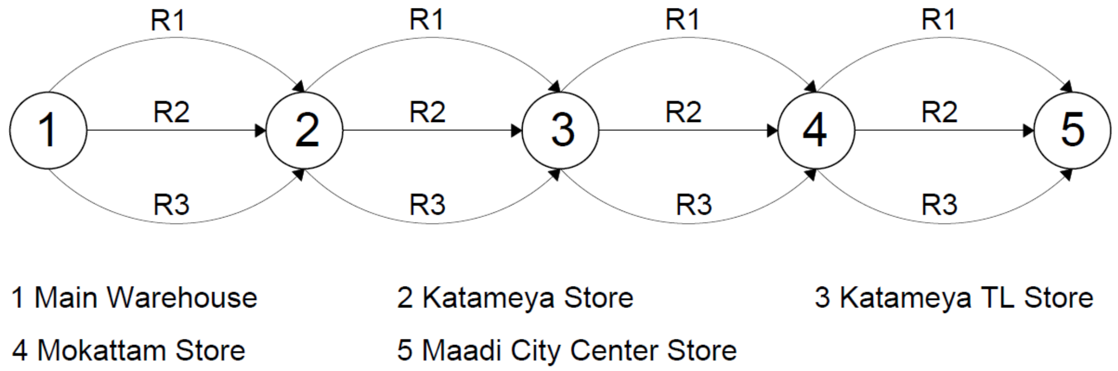

FIGURE 1. TRANSPORTATION NETWORK

Wen, Larsen, Clausen, Cordeau, Laporte have reviewed a special class of the vehicle routing problem with cross-docking. Cross-docking is a place where orders are to be consolidated but not to be stored for long period of time because the cross-dock is not equipped with inventory holding capability. Jayaraman and Ross (2003) presented the problem of deciding whether to open a cross-dock or not. They have proposed a tabu search algorithm to solve the problem. Lee, Jung and Lee (2006) have used the mixed integer programming to formulate the vehicle routing problem consisting of a single cross-dock, multiple suppliers and multiple customers. The vehicles leave the cross-duck, visit suppliers and or customers and return back to the cross-duck. The objective is to assign tours to a set of vehicles at the cross-dock so that suppliers and customers are visited within their time windows. They used a tabu search algorithm to solve the model.

Kenyon and Morton (2003) have considered the stochastic vehicle routing problem. They developed bounds on the objective function values and conditions to convert the problem from having stochastic to deterministic travel and service times. They considered two different models; the first minimizes the travel time from a head office until the vehicle returns to it after visiting all branches and the second maximizes the probability of visiting all branches by the specified time.

Li-Xia (2009) considered the multiple depot vehicle routing problems. He assumed a transportation network consisting M depots and N customers. Each depot has $V_i, i = 1, 2, \ldots, M$ vehicles. Each customer is served by only one vehicle of the M depot. He used a fuzzy simulation method to solve the problem.

Herazo-Padilla et al. (2013) considered the vehicle routing problem with stochastic travel speed and transportation cost. They used a technique based on both the discrete-event simulation and the ant colony optimization technique to find a solution to the problem. Ayadi, Chabchoub, and Yassine (2014) used a technique based on the branch and bound algorithm to find an exact solution to the vehicle routing problem of passenger's pick-up and delivery. Each customer should identify his origin and destination as well as desired departure and arrival times. Their research identifies the route to be assigned to each customer request taking into consideration the desired time constraints as well as the service quality.

Sambrani and Subhas used GIS to develop a Spatial Decision Support System (SDSS) for route analysis and route generation. GIS analyzes information and represents it visually. This visual representation allows for greater understanding of the operating environment around the corporate. Route analysis helps managers generate the shortest route that the vehicle should take between the company and their supply chain links. Alternative routes can also be generated and analyzed which help in rerouting of the supplies in case of problems on optimal route.

III. MODEL BUILDING

This section illustrates the development of the simulation model for the vehicle routing problem using the Pro-Model software. The transportation network consists of a warehouse and four branches: Katamia, Katamia T.L., Mokattam, and Maadi city center. Inter-arrival time to the warehouse, travel time between each two adjacent nodes, and loading or unloading times are each assumed to follow a probability distribution function. Company vehicles arrive to the warehouse (node 1) daily with probabilistic inter-arrival times to load merchandise and deliver them to the other four branches (nodes 2, 3, 4, and 5). Supplier vehicles as well as other vehicles may arrive to the warehouse to deliver merchandise.

The travel time measured in this research is the timeframe from when the company vehicle arrives at the warehouse until the vehicle completes the service at the last branch (node 5). At each branch, the vehicle loads and/or unloads materials with probabilistic service time distribution. Company vehicles visit the warehouse as well as some branches of which may result in waiting time for services at the warehouse and branches. After carefully studying the problem, 3 different routes connecting each 2 adjacent branches came to my attention. After considering all possible route combinations from node 1 to node 5, we studied 81 route alternatives. Sections 3.1 and 3.2 illustrate the process of analyzing and fitting theoretical probability distributions into input data; and sections 3.3 and 3.4 illustrate the simulation model building, verification, and validation.

3.1. Input Data Analysis

This section examines the collected data to ascertain their suitability for use in the simulation model. Several samples of data were collected: (1) Travel time data between adjacent nodes, (2) loading/ unloading times at each node, (3) vehicle inter-arrival times to warehouse, and (4) admin vehicle inter-arrival time to Katamia store. The data must be (1) independent; which is random and (2) identically distributed; the observations in a data set must have come from the same population.

3.1.1. Tests of Independence

Data are independent if the value of one observation is not affected by the value of another observation. Stat::Fit, statistical software bundled with the pro-model simulation software, is equipped with 3 tests for independence: (1) Scatter Plot, (2) Autocorrelation Plot, and (3) Runs Tests (Harrell, Ghosh, and Bowden, 2000). In the Scatter Plot, each two adjacent observations are plotted as one point. If the observations are dependent on each other, the scatter plot will form a trend line with slope depending on whether the correlation is negative or positive. The Autocorrelation Plot shows the correlation between observations. If the correlation coefficient is low; it means weak or no correlation, observations in the data set are random. Stat::Fit considers two types of runs tests: (a) Median test and (b) turning point test that test the hypotheses: H_0: Series is random; and H_1: Series is not random. The null hypothesis is to be rejected if the P-value is less than α, which is the identified level of significance.

3.1.2. Tests of Homogeneity

Data are considered to be identically distributed (non-homogenous) if the data in a single data set come from the same population. Testing the identically distributed property is referred to as the homogeneity test. Reasons of why data tend to be non-homogenous (Harrell, Ghosh, and Bowden, 2000):

a. When the simulation model has several entities and the service time of each entity follows a different probability distribution function.
b. When the time of the day impacts the inter-arrival time
c. When there are different causes of failure

Several ways are followed to test homogeneity of data: Visual inspection, and ANOVA tests for normally distributed data. If it is required to test two samples of data to see if they come from the same distribution, one may use Kolmogorov-Smirnov two-sample test or the Chi-square multi-sample test. The visual inspection test shows that either the observation in the data set have a single mode, single population; or they are bi- or multi-modal, the observation come from two or more populations.

3.2. Fitting a Theoretical Distribution to Input Data

Stat::Fit provides two Goodness-of-Fit tests (Harrell, Ghosh, and Bowden, 2000): (1) the Chi-square test and (2) Kolmogorov-Smirnov test. The goodness of fit techniques tests the null hypothesis H_0: The sample data come from a specific probability distribution; and the alternative hypothesis H_1: The sample data do not come from the specific probability distribution. Therefore, the result of the test of hypothesis can be used to disprove that the sample data come from a specified distribution but it cannot be used to statistically prove that the data come from that specific distribution.

Stat::Fit software was used to fit a probability distribution functions into each samples of data and the selected distributions are presented in Tables 1 and 2. The travel time distributions between adjacent nodes are presented in Table 1 and the service time distributions at each node are presented in Table 2.

TABLE 1. TRAVEL TIME DISTRIBUTIONS BETWEEN ADJACENT BRANCHES

Link	Travel Time distributions		
	Route 1	Route 2	Route 3
Warehouse to Katamia	Pearson5 (8, 134)	Lognormal (33, 1.8)	Lognormal (39, 1.62)
Katamia to Katamia T.L	Pearson5 (8, 1.78)	Lognormal (28, 1.31)	Triangular (78.5, 89, 102)
Katamia T.L. to Mokatam	Weibull (28, 1.32)	Lognormal (37, 1.31)	Lognormal (51.8, 2.22)
Mokatam to MCC	Weibull (23, 3.04)	Lognormal (38, 1.07)	Lognormal (4.19, 3.62)

TABLE 2. SERVICE TIME DISTRIBUTIONS AT NODES

Locations	Service time distributions
Warehouse	Lognormal (46, 2.64)
Katamia	Lognormal (78, 3.19)
Katamia T.L.	Lognormal (9, 1.36)
Mokattam	Gamma (23, 3.27)
Maadi City	Lognormal (47, 2.64)

3.3. Developing a Simulation Model

Building a simulation model using Pro-model software (Harrell, Ghosh, and Bowden, 2000) is presented in four concise steps: (1) build locations, (2) build entities, (3) build arrivals, and (4) build processing.

3.3.1. Build Locations

Locations refer to the fixed places in the simulation system where vehicles wait for service as well as the fixed places of which services are carried out. Due to the software limitation, 5 nodes were selected. Each node has 2 locations, one of them representing a queue for the waiting vehicles and the other representing the service area where vehicles are to be loaded or unloaded. The data required in this case are the locations name and capacity. For example, location 1 represents the waiting area for the warehouse while its capacity represents the number of spaces available for the waiting vehicles. Location 2 represents the loading /unloading areas capacity of which represents the number of servers (number of vehicles that can be served at the same time).

3.3.2. Build Entities

Entities represent the vehicles that use the transportation network i.e. company vehicles 1 and 2, agent vehicles, vender vehicles, and admin vehicles. Five different entities were considered in this simulation model, one for each type of vehicles. Company vehicle 1 is the only vehicle that travels from node 1 to node 5. The other vehicles use only part of the transportation network. Company vehicle 1 loads merchandise from the warehouse (node1) and delivers them to all branches (nodes 2-5) then exits the simulation system. Company vehicle 2 visits the warehouse (node 1) to deliver merchandise then exits the system. The agent vehicles visit the warehouse (node 1) to load/unload merchandise then exit the system. Vender vehicles visit the warehouse (node 1) to deliver merchandise then exit the system. Admin vehicles visit the Katamia branch location then exit the system. Then simulation model only keeps track of the travel and waiting times of store vehicle 1. Other vehicles only affect the waiting time of company vehicle 1.

3.3.3. Build Arrivals

Build arrival exemplifies how often each type of vehicles arrives to the simulation model (warehouse) and how often admin vehicles arrive to the Katamia branch. In this step, we feed the Pro-model simulation program with the inter-arrival probability distribution functions related to each entity.

3.3.4. Build Processing

Build Processing describes the operation taking place at each location in the system. It explains in details the order of waiting, processing activities and exiting the system. In this case study, we will develop a simulation model for each possible route, i.e. we will develop 81 different simulation models. For example, if we consider route R-1111; the order of processing will be as follows. The initial step would be to arrive at waiting area of warehouse (location 1) and remain until the server is ready. Then, the server will then load the merchandise (location 2) from the warehouse with a service time of which follows Lognormal (46, 2.64) (in minutes). The company vehicle then travels from the warehouse to Katamia branch (location 3) with travel time of which follows Pearson5 (8, 13.4) (in minutes). The company vehicle will then wait at Katamia waiting area (location 4) and unload the merchandise therein with a service time of which follows Lognormal (9, 1.36) (in minutes). The company vehicle will complete the route respectively until it reaches node 5 (location 10), then it will exit the system.

3.4. Model Verification and Validation

The process of verification is performed as a testing tool to ensure that the simulation model has no particular errors. The validation of the model must assure that the model imitates the actual transportation systems behavior.

IV. MODEL SOLUTION AND STATISTICAL ANALYSIS OF RESULTS

This section presents the results generated from the simulation model containing only company vehicle 1. The other vehicles are not considered in this section and will be added to the model in the sensitivity analysis section.

4.1. Model Solution

After the model verification and validation are done, a sample size of 30 replications was planned to have higher result accuracy. A warm up period was considered to prevent the model from collecting data at the beginning of the simulation before reaching the steady state conditions. The model collects data about the travel time of vehicle 1 from the time it enters the system (arrive to the warehouse) until the time it exists the system (complete service at node 5). These simulation results were repeated 30 different times for each of the 81 different routes. Table 3 shows the averages and standard deviations of the travel time for the 30 replications at each possible route.

TABLE 3. SIMULATION RESULTS

Route	Average	Stdv	Route	Average	Stdv	Route	Average	Stdv
R-1111	339.38	39.04	R-2111	375.16	43.38	R-3111	385.16	39.09
R-1112	366.47	31.62	R-2112	403.90	44.19	R-3112	411.90	36.01
R-1113	339.11	36.55	R-2113	389.08	30.53	R-3113	379.99	30.77
R-1121	381.24	42.15	R-2121	408.18	36.38	R-3121	418.78	35.99
R-1122	397.15	37.40	R-2122	435.82	42.24	R-3122	451.90	32.85
R-1123	367.88	38.83	R-2123	435.82	42.24	R-3123	451.90	32.85
R-1131	410.13	47.42	R-2131	422.26	39.59	R-3131	428.22	38.79
R-1132	432.14	38.31	R-2132	465.48	30.98	R-3132	463.73	45.66
R-1133	385.83	44.53	R-2133	427.49	35.95	R-3133	424.70	35.11
R-1211	373.49	44.73	R-2211	402.12	38.46	R-3211	400.71	38.89
R-1212	393.26	43.27	R-2212	434.51	34.12	R-3212	437.86	42.90
R-1213	360.34	31.31	R-2213	396.62	35.09	R-3213	418.92	44.95
R-1221	399.48	35.97	R-2221	432.38	40.11	R-3221	447.43	29.03
R-1222	432.63	42.27	R-2222	474.30	40.34	R-3222	483.33	43.25
R-1223	413.51	45.89	R-2223	427.75	35.75	R-3223	440.76	36.19
R-1231	425.44	38.82	R-2231	445.98	31.56	R-3231	455.39	42.87
R-1232	448.78	34.78	R-2232	486.49	31.31	R-3232	503.62	38.96
R-1233	420.79	45.17	R-2233	440.50	29.13	R-3233	474.17	43.37
R-1311	435.03	36.98	R-2311	470.09	27.02	R-3311	466.20	42.44
R-1312	475.06	49.04	R-2312	498.87	37.18	R-3312	507.72	39.69
R-1313	418.66	28.61	R-2313	462.92	35.30	R-3313	471.21	32.01
R-1321	466.89	37.69	R-2321	503.51	41.07	R-3321	497.85	33.18
R-1322	494.67	36.23	R-2322	535.03	43.37	R-3322	537.16	38.51
R-1323	469.81	55.37	R-2323	503.04	38.61	R-3323	504.81	38.18
R-1331	479.91	44.42	R-2331	508.69	34.12	R-3331	479.91	44.42
R-1332	513.32	37.26	R-2332	553.71	36.44	R-3332	513.32	37.26
R-1333	483.76	38.56	R-2333	500.13	31.64	R-3333	524.13	42.32

4.2. Statistical Analysis of Simulation Results

This section illustrates the analysis of the simulation results to determine the optimal routes with the lowest average travel time and develops confidence intervals.

4.2.1. Optimal Route Selection

This section provides an output data analysis for the simulation results to select the optimal set of transportation routes with minimum travel time.

Transportation routes shown in Table 3 were sorted based on their average travel time. Routes R-1113, R-1111, R-1213, R-1112, and R1123 have the lowest averages of travel time in the list. Analysis of variance (ANOVA) (Lind, William, and Wathen, 2008) was employed to test the possibility of having significant differences between the travel time averages of the selected routes. The null and alternate hypotheses were stated as: H_0: $\mu_{R\text{-}1113} = \mu_{R\text{-}1111} = \mu_{R\text{-}1112} = \mu_{R\text{-}1123} = \mu_{R\text{-}1213}$ and H_1: Not all the means are equal.

The result of a single factor ANOVA test presented in Table 4 showed that the null hypothesis is to be rejected since the P-value of this test was close to zero. Therefore the travel time averages taken when using the identified routes are not equal; the five selected routes have significant differences in their travel time.

We reduced the five routes to two routes: R-1111 and R-1113, and repeated the test. The new hypothesis statement is: H_0: $\mu_{R\text{-}1113} = \mu_{R\text{-}1111}$ and H_1: Not all means are the equal. The ANOVA test presented in Table 5 shows that the P-value is 0.98. Therefore, we fail to reject H_0. Hence, there is no significant difference between the mean travel times on routes: R-1111 and R-1113. Therefore, routes R-1111 and R-1113 are selected to be the routes with the shortest travel time.

TABLE 4. SINGLE FACTOR ANOVA

SUMMARY				
Groups	Count	Sum	Average	Variance
R-1113	30	10,173.20	339.11	1,336.05
R-1111	30	10,181.43	339.38	1,524.07
R-1213	30	10,810.12	360.34	980.05
R-1112	30	10,993.96	366.47	999.78
R-1123	30	11,036.41	367.88	1,508.03

ANOVA						
Source of Variation	SS	DF	MS	F	P-value	F Crit
Inter-Group	24,651.44	4	6,162.86	4.85	0.00	2.34
Intra-Group	184,091.39	145	1,269.60			
Total	208,742.83	149				

TABLE 5. SINGLE FACTOR ANOVA

SUMMARY				
Groups	Count	Sum	Average	Variance
R-1113	30	10,173.20	339.11	1,336.05
R-1111	30	10,181.43	339.38	1,524.07

ANOVA						
Source of Variation	SS	DF	MS	F	P-value	F Crit
Inter-Group	1.13	1	1.13	0.0	0.98	4.01
Intra-Group	82.943.53	58	1,430.06			
Total	82,944.66	59				

4.2.2. Confidence Intervals of the Mean Travel Time

This section illustrates the development of the confidence intervals of the mean travel time at routes R-1111 and R-1113. Since the population standard deviation σ is unknown, the t distribution is used to determine the confidence interval as shown in the following equation (Lind, William, and Wathen, 2008):

$$\bar{x} \pm t_{\alpha/2} \frac{S}{\sqrt{n}}$$

If the 95% level of significance is chosen, the corresponding value of $\alpha = 0.05$, the tabulated value of $t_{\alpha/2} = 2.045$ at 29 degrees of freedom. The confidence intervals for the average travel times on the optimal routes shown in Table 6 are (324.8 to 353.96 minutes) and (325.46 to 352.76 minutes). Therefore, the mean travel time of the population for route R-1111 is likely to occur within the range 324.8 and 353.96 whereas that of route R-1113 is likely to occur within the range 325.46 and 352.76 with 95% probability.

TABLE 6. CONFIDENCE INTERVALS FOR THE AVERAGE TRAVEL TIMES ON THE OPTIMAL ROUTES

Optimal Routes	95% Confidence Intervals	
	Lower Limit	Upper Limit
R-1111	324.8 minutes	353.96 minutes
R-1113	325.46 minutes	352.76 minutes

V. SENSITIVITY ANALYSIS

This section studies two cases. The first case considers the transportation network to include company vehicle 1 only. It presents a study of how the changes in the travel time variability at each link of the transportation network affect the overall travel time and variability. In addition, it shows how these changes affect the selection of the optimal transportation route.

The second case considers the presence of other vehicles along with company vehicle 1 in the transportation network. It shows how the presence of other vehicles in the system affects the overall travel time and variability. In addition, it illustrates their effect on the selection of the optimal transportation route.

5.1. Changes of Travel Time Variability

This section considers the simulation model previously described with company vehicle 1 only using the system; consisting of the 5 routes that were found to have the shortest travel time. Three levels of standard deviations of the travel time at each link were considered: 10%, 20% and 30% of the average travel time. The simulation model was executed for 30 replications per level of standard deviation on each of the selected routes. Simulation will collect the necessary data to compute both the average travel time and standard deviation for each case. Table 7 shows the simulation results at each level of standard deviation. Results show that as the values of the travel time standard deviation increase at all links in the network, the average travel times also increase. When the travel times' standard deviations increase by 10%, at all links, the average travel time of the entire trip would increase by more than 12%. Results also show that as the standard deviations increase at all links, the standard deviation of the entire trip may or may not increase. Therefore, the relationship between the standard deviations at all links and that of the entire trip has no specific trend.

TABLE 7. IMPACT OF THE STANDARD DEVIATION CHANGE ON THE AVERAGE AND STANDARD DEVIATION OF TRAVEL TIMES

Standard deviation of distribution average		10%	20%	30%
Route 1113	Average	328.3	378.6	431
	Standard dev.	39.2	38.1	33.2
Route 1111	Average	318.5	385.9	429.2
	Standard dev.	42.2	44.9	32.99
Route 1213	Average	355.9	404.9	450
	Standard dev.	55.5	40.9	32.4
Route 1112	Average	364.16	406.1	463.2
	Standard dev.	34.5	39.5	41.4
Route 1123	Average	358.3	410.8	454.7
	Standard dev.	41.6	39.3	28.9

Results also show that as we change the standard deviations at each link, the optimal route slightly changes. When we used the actual data, Route 1113 was identified to be the one with the shortest travel time. As we changed the standard deviations at all links to be 10% of average travel time, Route 1111 was the one selected. As we increased the standard deviation to 20% and 30% of the average travel time at these links, Route 1113 was identified to be the route with the shortest travel time again.

5.2. Presence of Other Vehicles in the System with the Company Vehicles

When other supplier vehicles are present in the system, they will impact the company vehicles' waiting time. Other supplier vehicles in the simulation model were permitted to use service areas at both the head office and branches then exit the system. Since several vehicles use the system at the same time, they impact the company vehicles' waiting time which affects the overall travel time. Table 8 shows that when other vehicles enter the system, the route with the shortest travel time slightly changes from Route 1113 to Route 1111. Results show that the presence of other vehicles in the system increases the travel time of company vehicle 1. The increase in travel time depends on the number of vehicles in the system as well as the service time needed for each vehicle.

VI. CONCLUSIONS

This research considers the problem of selecting the optimal transportation route of a transportation network with probabilistic travel and service times. We collected data about the vehicle inter-arrival time, travel time at links, and service time at various branches and the warehouse. We developed a simulation model to determine the travel time at all possible routes. Results revealed that routes R-1111 and R-1113 have the shortest travel time.

Sensitivity analysis was conducted to find the impact of changing the standard deviations of travel time at each link on the overall average and standard deviations of travel time. Results show that as the standard deviation increases, the average travel time increases. It also shows that due to the increase of the standard deviation of travel time at the links, the optimal transportation route may slightly change. When other vehicles enter the system, the average trip time increases. It also leads to the change of the optimal transportation route.

TABLE 8. TRAVEL TIMES BEFORE AND AFTER ADDING OTHER CARS

Route	Company vehicles as well as other vehicles use the system	Company vehicle only
Route 1113	339.11	328.26
Route 1111	339.38	318.47
Route 1213	360.34	355.89
Route 1112	366.47	364.16
Route 1123	367.88	358.29

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