A Fuzzy Set and Rule-Based Approach to Guide Supplier Selection Decisions

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A fuzzy-set based approach is developed that first considers the fuzzy probabilities of a set of suppliers satisfying attributes for selection within the criteria designated as Delivery, Front Office Quality and Value-Added Services. Based upon the expected values for each attribute for each supplier, the algorithm develops a rule-based approach for selection practices. The degree of certainty of the decision rule is set after consideration of a maximum and minimum score to which the respondents' scores are either contained in or intersect the decision rules. These membership values are shown to be relevant to statistical confidence. An application is presented from an extensive survey database for six dominant suppliers in the targeted market. The results demonstrate important supplier considerations beyond price, delivery and quality affect and drive supplier selection decisions. Discussion of the results and conclusions about the validity and robustness of the methodology to supplier selection are presented.

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I. INTRODUCTION

The fuzzy rule-based model proposed in this study effectively and efficiently captures alternative criteria for supply chain decision makers. Measuring the performance of suppliers (Simpson, Siguaw, and White, 2002) and stressing the importance of supplier selection criteria (Vijayvagy, 2012) for small firms in the United States (Park and Krishnan, 2001) or large firms in Japan (Hirakubo and Kublin, 1998) reinforces that this initiative is not restricted to large firms and is global. Typical criteria used are price, delivery, and quality (PDQ) (Hirakubo and Kublin, 1998; Simpson, Siguaw, and White, 2002), but Park and Krishnan (2001) add several managerial criteria beyond PDQ including managerial forecasts, trust level, and organizational structure. Supplier selection research using decision support systems also extends supplier selection criteria in other dimensions including environmental considerations (Handfield, Walton, Sroufe, and Melnyk, 2002). Weber, Current and Benton (1991) consider other factors like facility location, capacity, and financial position. Ellram (1990) discusses the importance of incorporating these and other

decision criteria in the process of selecting suppliers, and Simpson, Siguaw and White (2002) provide an extensive study of criteria used by companies on their supplier assessment forms. Using the traditional PDQ criteria, Verma and Pullman (1998) provide an extensive study of how purchasing managers evaluate trade-offs among the criteria. They point out that, while the PDQ criteria are generally accepted in industry, delivery and quality lend themselves to multi-criteria decision support models because the complexity and varied dimensions of delivery and quality can confound a decision maker. Of these criteria, quality tends to be extensively studied by researchers (Liu and Hai, 2005), although supplier selection and evaluation is perceived to be most often based entirely on price (Degraeve and Roodhooft, 1999, 2001).

Alternatives for supplier selection decisions range from simple expert opinion methods to quantitative methods. Even where quantitative supplier performance data are readily available, subjective judgment of qualitative performance metrics must be provided by a variety of sources, including senior management (Ghodsypour and O'brien, 1998; Humphreys, Mak, and Yeung, 1998; Verma and Pullman, 1998), experts in the field (Cheng, Li, Love, and Irani, 2001; Humphreys, Shiu, and Chan, 2001; Mandal and Deshmukh, 1994; Rebstock and Kaula, 1996) and even a project's team members (Ragatz, Handfield, and Scannell, 1997). Often, supplier selection models have focused on using the Analytic Hierarchy Process (AHP) or providing case study illustrations of decision making processes to address the need for utilizing expert opinions (Guneri and Kuzu, 2009; Hadi-Vencheh and Niazi-Motlagh, 2011; Liu and Hai, 2005). However, subjective judgments offered in terms of linguistic variables provide a degree of uncertainty and insert ambiguity into the decision. Customer demands are generally uncertain and supplier evaluation, selection and coordination lead to various strategies to manage supplier relationships (Chan, 2003; Deng and Elmaghraby, 2005). Fuzzy logic has been recognized as an important tool in the analysis of uncertainty in decision making situations, including supply chain management (SCM).

Lui (1999), proposed a fuzzy model for partial backordering models in 1999. Little was done with inventory considerations until fully five years later when inventory discounting considered the buyer-seller relationships (Das, Roy, and Maiti, 2004), and location aspects for inventory control became fuzzy considerations (Usenik and Bogataj, 2005). Supply chain decisions for integrated just-in-time inventory systems recognized the fuzzy nature of annual demand and production rates as being no longer statistically based. Fuzzy annual demand and/or production rate offered an answer by employing the signed distance, a ranking method for fuzzy numbers, to estimate fuzzy total cost of the JIT production in today's supply chain environment. A fuzzy-set based method derived the optimal buyer's quantity and number of lots from the vendor (Pan and Yang, 2007). Later, Lin (2012) combined fuzzy methods with linear programming to optimize order allocation under uncertainty.

Fuzzy programming contributed to the following: optimal product mix based on ABC analysis (Kara, Gökçen, and Atasagun, 2009): fuzzy multi-objective linear programming minimized total production and transportation costs; the number of rejected items and total delivery time as related to labor and budget constraints (Liang, 2007); and fuzzy goal programming considered supply chain management from the perspective of activity-based costing with mathematically derived optimization for evaluating performance of the value-chain relationship (Tsai and Hung, 2009). Manufacturing processes as related to business logistics looked at the data itself as fuzzy in Quality Function Deployment's relationship to customer service (Shu and Wu, 2009). The attainment of goals such as quality further led

to attempts to balance production processes of assembly lines. Fuzzy goals were used as an instrument and product for measuring, displaying and controlling industrial process variables (Kara, Gökçen, and Atasagun, 2009).

Considering different quality standards in a supply chain network a fuzzy neural approach was utilized to suggest adjustments of product quantity from various suppliers (Chan, Kumar, Tiwari, Lau, and Choy, 2008). The Fuzzy Suitability Index (FSI) aggregated rankings and multiplied, by weight, each criterion (Bevilacqua and Petroni, 2002). With the same goal of ranking suppliers according to performance, a method was proposed whereby n decision makers evaluated the performance of *m* suppliers in *k* criteria, rating the importance of the *k* criteria in linguistic terms. Aggregation of the fuzzy expressions for importance weights, and a fuzzy preference index led to rank ordering of the suppliers (Bayrak, Celebi, and Taşkin, 2007). Amid, Ghodsypour, and O'Brien (2011) demonstrated the use of the fuzzy weighted min-max to select suppliers using AHP as a component of criteria weighting. Ishizaka (2014) later blended AHP and fuzzy logic using the Hybrid Fuzzy AHP method in an attempt to harness the advantages of both. Chen (2011) combined the fuzzy TOPSIS method with several other decision methods to rank order suppliers on the basis of criteria preferences and a SWOT analysis. Ferriera and Borenstein (2012) coupled fuzzy logic with Bayesian networks and influence diagrams to evaluate and rank suppliers.

Besides the concept of rank ordering based on fuzzy scores, an approach of interest to this research's premise is fuzzy associated rule mining from a database for supplier assessment (Jain, Wadhwa, and Deshmukh, 2006). Lastly, as justification for a comprehensive fuzzy set and rule-based model, Sevkli (2009) in his comparison of a recognized crisp ELECTRE model versus a fuzzy ELECTRE model, concluded that using fuzzy sets for multi-criteria supplier selection decisions is superior. The work presented herein reinforces this perspective by using a fuzzy rule-based approach to capture the importance of alternative criteria of decision makers without relying on strictly statistical processes and rank ordering output. Instead, the model provides decision alternatives that move beyond pair-wise comparisons of attributes by assessing levels of membership in qualitatively defined parameters and certainty of beliefs in the generated rules.

II. FUZZY SUPPLIER SELECTION MODEL

The following concepts are necessary for the algorithmic development that follows.

2.1. Fuzzy Set Basics

Fuzzy logic addresses the ambiguity of data and uncertainty in a decision making situation, where a fuzzy subset *A* of a set *X* is a function of *X* into [0,1]. For a brief foundation in the basics, see (Bellman and Zadeh, 1970; Dubois and Prade, 1980; Freeling, 1980; Zadeh, 2006). While a new class of implication operators has been proposed (Yager, 2004), the more traditionally utilized fuzzy operations are used in this research. *A* and *B* denote two fuzzy sets, so the intersection, union, and complement are defined by:

$$A \cap B = \Sigma\, \gamma_i / x_i, \text{ where } \gamma_i = \text{Min}\, \{\alpha_i, \beta_i\}; \quad (1)$$

$$A \cup B = \Sigma\, \gamma_i / x_i, \text{ where } \gamma_i = \text{Max}\, \{\alpha_i, \beta_i\}; \quad (2)$$

$$\neg A = \Sigma\, \gamma_i / x_i, \text{ where } \gamma_i = 1 - \alpha_i; \quad (3)$$

and it is assumed that $B = \Sigma\, \beta_i / x_i$ (Kaufmann and Gupta, 1985; Klir and Folger, 1988; Zadeh, 1965; Zadeh, 1975).

Extension principles (Dubois and Prade, 1980; Gupta, Ragade, and Yager, 1979; Zebda, 1984) often guide the computations when

dealing with fuzzy sets. Letting f be a function from X into Y, with Y as any set and A as above, then f can be extended to fuzzy subsets of X by:

$f(A) = \Sigma_y \, u_{f(A)}(y) / y$, where
$u_{f(A)}(y) = \text{Max}_{x \varepsilon f^{-1}(y)} A(x)T$ (4)

Thus, $f(A)$ is a fuzzy subset of Y. In particular, if f is a mapping from a Cartesian product such as $X \times Y$ to any set, Z, then f can be extended to objects of the form (A,B) where A and B are fuzzy subsets of X and Y by:

$f(A,B) = \Sigma \, u_{f(A,B)}(z) / z$, where
$u_{f(A,B)}(z) = \text{Max}_{(x,y)\varepsilon f^{-1}(z)} \text{Min}\{A(x), B(x)\}.$ (5)

A fuzzy set P whose elements all lie on the interval [0,1] can be expressed as a fuzzy probability. Consider a set of n fuzzy probabilities each having r elements,

$$a_i = \sum_{j=1}^{r} \alpha_{ij} / a_{ij} \ \text{ for } i = 1, 2, \ldots, n,$$ (6)

where α_{ij} denotes the degree of belief that a possible value of a_i is a_{ij}. Then $(a_1, a_2, \ldots a_n)$ constitutes a finite fuzzy probability distribution if and only if there are n-tuples a_i, $i = 1,2,\ldots,n$ such that $\sum_{i=1}^{n} a_i = 1$. Consider a set of n fuzzy probabilities each having r elements,

$$p_i = \sum_{j=1}^{r} a_{ij} / p_{ij} \ \text{ for } i = 1, 2, \ldots, n, \text{ where } a_{ij}$$

denotes the degree of belief that a possible value of p_i is p_{ij}. Then $(p_1, p_2, \ldots p_n)$ constitutes a finite fuzzy probability distribution if and only if there are n-tuples p_i, $i = 1,2,\ldots,n$ such that $\sum_{j=1}^{n} p_i = 1$.

To qualify as a finite fuzzy probability distribution, each fuzzy probability in the distribution must have the same number of elements (some of the a's may be zero), and these elements should be ordered in the sense

that the sum of the elements in each specific position must equal one. So the n-tuples (a_{ij}), $i=1,2,\ldots,n$ form probability distributions in the crisp sense. This type of probability distribution can be transformed such that the resulting distribution has entropy at least as great as the original (Yager and Kreinovich, 2007).

A version of fuzzy expected values was first used when Zebda (1984) defined $Q_{ijk} = \Sigma \, \alpha_{ijk} / a_k$ as the fuzzy probability that from State i and making Decision j, reach State k. Associated with this are fuzzy benefits B_{ijk} where $B_{ijk} = \Sigma \, \text{ß}_{ijk} / b_k'$. Then the averaged benefit is defined by $E(B_{ijk}) = \Sigma c_{ij\ell} / b_\ell$ where:

$c_{ij\ell} = $
$\text{Max}(a_1, \ldots a_p, b_1', \ldots b_p') \varepsilon f^{-1} b_\ell k \, \text{Min}(\alpha_{ijk}, \text{ß}_{ijk}),$ (7)

for $b_\ell = \Sigma_k \, a_x b_x$ if $\Sigma_k \, a_x = 1$ and 0 otherwise. Here, $f(a_1, \ldots, a_p, b_1', \ldots, b_p') = \Sigma \, a_x b_x'$.

2.2. Rough Set Notation Applied to Fuzzy Sets

Considering a fuzzy subset A of U, as the Universe of Discourse, is defined by a characteristic function $\mu_A : U \rightarrow [0,1]$, the notation $\Sigma \, \alpha_i / x_i$ $(0 \le \alpha_i \le 1)$ denotes a fuzzy subset whose characteristic function at x_i is α_i. Following the previous discussion of fuzzy operators, if A and B are fuzzy subsets, $A \cap B$, $A \cup B$, and $\neg A$ are defined by Min $\{\mu_A(x), \mu_B(x)\}$, Max $\{\mu_A(x), \mu_B(x)\}$, and $1 - \mu_A(x)$, respectively. The implication $A \rightarrow B$ is defined by $\neg A \cup B$. The corresponding characteristic function is Max $\{1 - A(x), B(x)\}$.

Two functions of pairs of fuzzy sets that will be used to determine rules for selecting a supplier defined as:

$I(A \subset B) = \inf_x \text{Max}\{1 - A(x), B(x)\},$ (8)

$J(A \# B) = \text{Max}_x \text{Min} \{A(x), B(x)\}.$ (9)

Here A and B denote fuzzy subsets of the same universe. The function $I(A \subset B)$ measures the degree to which A is included in B and $J(A \# B)$ measures the degree to which A intersects B. Indeed, if A and B are crisp sets it is easy to establish that $I(A \subset B) = 1$ if and only if $A \subset B$; otherwise it is zero. Also, in the case of crisp sets $J(A \# B) = 1$ if and only if $A \cap B \neq \varnothing$; otherwise it is zero. It is also clear that I and J can be expressed as

$$I(A \subset B) = \inf_{x} (A \rightarrow B), \qquad (10)$$

$$J(A \# B) = \text{Max}_{x} (A \cap B). \qquad (11)$$

In addition, the following relation holds:

$$I(A \subset B) = 1 - J(A \# \neg B). \qquad (12)$$

The operators I and J will yield two possible sets of rules: the certain rules and the possible rules. The primary objective is to see to what degree a combination of attributes is a subset of the decision (certain rules) or intersects the decision (possible rules) to select a supplier. The specific computations are in the Application section.

III. ALGORITHM

The algorithm preserves information during the process of computing and evaluating fuzzy probabilities then defuzzifying the data into a score (Steps 0-6). The relationship of the fuzzy set-based defuzzified score then contributes certain and possible rules for supplier selection under varying combinations of attribute (Steps 7-10).

0. Randomly partition the criteria data set into ℓ subsets of equal size.
1. For each attribute ϕ of each supplier v, subjectively assign scores $s_{\phi k v}$.

The supplier rating ($s_{\phi k v}$) is then given by the equation $s_{\phi k v} = \sum \tau_{\phi k} / s_{\phi k}$ for all v where $\tau_{\phi k} = 1$ ($v=1,2,\ldots,m$; $k=1,2,\ldots,n$; and $1 < \phi < x$).

2. Define the fuzzy expected value, $Q_{\phi k v}$, for each attribute ϕ of each v in terms of each $s_{\phi k v}$ as $Q_{\phi k j v} = \sum \alpha_{\phi k j v} / a_{\phi k j v}$ for all $s_{\phi k j v}$, where each $\alpha_{\phi k j v}$ represents belief in the probability $a_{\phi k j v}$ that v will be scored $s_{\phi k j v}$ ($v=1,2,\ldots,m$; $k=1,2,\ldots,n$; $1 < \phi < x$ and $j=1,2,\ldots,\ell$).

3. Group the probabilities $a_{\phi k j v}$ into combinations $\varphi_{\phi v}$ such that $\sum a_{\phi k j v} = 1$ for some set H of k's. $a_{\phi k j v} = 0$ for $k \notin H$.

4. Across all partitions ℓ, compute $b_{\phi v} = \{\sum a_{\phi k j v} s_{\phi k j v}$ if $\sum a_{\phi k j v} = 1$, otherwise 0 ($k = 1,2,\ldots,r$; $j=1,2,\ldots,\ell$ and $p =$ the distinct number of $\sum a_{\phi k j v} = 1$; $1 < \ell \leq p$).

5. For all $\alpha_{\phi k j v} \neq 0$ find $c_{\phi v} = \text{Min} \{\tau_{\phi k j v}, \alpha_{\phi k j v}\}$, where $c_{\phi v}$ is the degree of belief that the expected value is $b_{\phi v}$.

6. Defuzzify the expected value for each attribute ϕ to find $E(s_{\phi v}) = \sum c_{\phi v} b_{\phi v} / \sum c_{\phi v}$

7. Compare the defuzzified score for each supplier by attribute relating the score to the maximum and minimum values over the set of all suppliers for that attribute; up to $\pm 3\sigma$

8. Determine I and J functions, where by (10) and (11):
   $I(A \subset B) = \inf_{x} \text{Max} \{1 - A(x), B(x)\}$,

   $J(A \# B) = \text{Max}_{x} \text{Min} \{A(x), B(x)\}$.

9. Set level of acceptance for the rules; Ω

10. Determine combinations of certain and possible rules for selecting a supplier based on Ω where rules are of the form:

"If Attribute ϕ_{k1} is {High, Low} and Attribute ϕ_{k2} is {Great, Small} for criterion j over each v in terms of each $s_{\phi kv}$, then select supplier 1,2,…,m."

IV. APPLICATION

The algorithmic example uses results from a survey instrument built with input from industry expert focus groups. The subsequent survey measures customer ratings of a group of suppliers for various variables including delivery attributes.

4.1. Example Data

The survey was distributed to about 3,000 companies that purchase semi-conductors, passives, RF/microwaves, connectors and interconnects, and electromechanical devices from a small set of dominant suppliers. Representative industries included automotive, communications, contract design/engineering, power/electrical, medical/dental, computer, manufacturing, and military/aerospace. The survey queried each customer's number of years of activity in the industry in designated ranges from less than two to 21 or more. Customers dealt with multiple suppliers and specified their firm's annual sales revenue as under $5,000,000 to over $2,000,000,000. With 412 surveys received, the response rate was slightly under 15%.

Delivery specific questions related to on-time performance, availability of inventory, shipping accuracy and return authorization process were queried in 20, 21, 22, and 27 (Handfield, Walton, Sroufe, and Melnyk, 2002). Front office quality was assessed based on quote completeness, credit and payment terms, cancelled/non-returnable letters, and contract processing (questions 23, 25, 26 and 28) (Liang, 2007; Tsai and Hung, 2009). Finally, value-added, support- specific quality assessment dealt with customized processing, knowledgeable specialists, technical design services, e-business services and sales management (questions 24, 29, 30, 31 and 32) (Chan, Kumar, Tiwari, Lau, and Choy, 2008). Price considerations were captured on quote criteria (Degraeve and Roodhooft, 1999; 2001).

An extensive study by Simpson, Siguaw, and White (2002) determined the highest number of forms and percentage of all forms containing the typical Price, Delivery, and Quality (PDQ) with other selection criteria. To validate the survey's relevance, a comparison was made as shown in Table 1 below.

TABLE 1. CORRESPONDENCE OF SURVEY TO USAGE IN INDUSTRY

Survey Category	Industry Question	Simpson, Siguaw and White	% of Forms
Delivery	On-time delivery	Delivery timeliness	61.9
	Availability of inventory	Inventory accuracy Fill Rate	15.5 15.5
	Shipping accuracy	Accurate delivery Inspection Prior to Shipping	32.1 27.4
	Return material authorization	Return procedures Complaint Handling Process	20.2 33.3
Front Office Quality	Quote completeness & turnaround	Quality documentation Prompt ordering process Timely ordering	48.8 28.6 16.7
	Credit and payment terms	Payment process	10.7
	Non-cancellable-return letters	Corrective/preventative measures	54.8
	Contract processing	Customer/PO requirements met Accurate invoicing	78.6 20.2
Value Added Services	Knowledge specialists adding value	Staff problem solver Staff expertise	11.9 20.2
	Technical design services	Technical assistance	32.1
	e-business service	EDI capability Inventory mgt. system Inventory tracking	14.3 35.7 35.7
	Sales & sales management support	Quality management Staff responsive	54.8 21.4
	Customized Processing	Segregation of nonconforming product Staff flexibility/Cooperative Open Idea Generation Process	46.4 10.7 7.1

Thus, Delivery, Front Office Quality and Value-Added Services became the categories into which the survey attributes for supplier selection were grouped. The respondents were asked to score the attributes on a 0–5 Likert scale for seven suppliers: Arrow, Avnet, Future, Insight, Kent, Pioneer and TTI. For model application purposes, the survey provided performance measurements on each supplier, as well as measures of the importance of each criterion to the customer and the customer's level of belief explicitly tied to the company's annual amount of business conducted with the targeted group of suppliers. Survey questions relating directly to the importance of this fuzzy supplier selection application included a query of the amount of money the customer spends on electronic components in a year. These ranges were: <$100,000; $100,000..$499,999; $500,000..$999,999; $1,000,000..$9,999,999; $10,000,000..$24,999,999; and >$25,000,000.

These ranges were used to identify a firm's level of activity with the suppliers in question and, therefore, its expected level of confidence (interpreted as r belief) in its assessments.

Some respondent data could not be considered because of incomplete responses. In addition, one supplier, Kent, was removed from the set due to: a) low survey responses compared to the other suppliers, and b) no longer existing as an independent company, having been acquired by Avnet after the survey was conducted. The resulting dataset left a pool of 150 useful responses to be applied to the fuzzy algorithm.

The remaining survey responses were randomly partitioned into two sets of 75 responses each in accordance with Step 0 of the model algorithm. These respondents evaluated suppliers on the delivery-specific attributes: on-time performance, availability of inventory, shipping accuracy and RMAs.

4.2. Modeling Process

By Step 1 of the algorithm, $\phi = 1,2,3,4$ attributes as defined above. Each of the four attributes is subjectively assigned a score by the respondent for each of the six suppliers ($m=6$), equating to Poor, Below Average, Average, Above Average and Excellent ($n=5$). Supplier rating $s_{\phi v}$ is then given by the equation $s_{\phi v} = \sum \tau_{\phi k v} / s_{\phi k v}$ for each supplier, v, and, by Step 2, the fuzzy probability $Q_{\phi k j v}$, for each attribute of v in terms of $s_{\phi k j v}$ is $Q_{\phi k j v} = \sum \alpha_{\phi k j v} / a_{\phi k j v}$ for all $s_{\phi v}$. Each $\alpha_{\phi k j v}$ represents belief in the probability $a_{\phi k j v}$ that v will perform to the level of the assigned score $s_{\phi v}$ ($k=1,2,\ldots,5$; $v =1,2,\ldots,6$; $\phi =1,2,3,4$; and $j=1,2$).

The belief functions were populated based on a survey question indicating the amount of annual spending done by the respondent. Table 2 describes the scoring of respondent belief as proportional to total possible spending (conservatively assumed to be the low end of the top category, $25,000,000):

At this point, a spreadsheet was used to organize and solve the equations. The expected values are calculated based upon the algorithmic steps. For example, after the assignment of belief, the algorithmic process for supplier v shows four significant digits to make the method clear, although subsequent suppliers are rounded to two significant digits for readability and brevity. $Q_{\phi k v}$ is as follows:

TABLE 2. RESPONDENT BELIEF ASSOCIATED WITH SPENDING

Spending	Degree of belief
< $100,000	0.0020
< $500,000	0.0100
< $1,000,000	0.0300
< $10,000,000	0.2000
< $25,000,000	0.7000
> $25,000,000	1.0000

$Q_{111} = 0.0000/0.0000 + 0.2500/0.0400$ for $s_{\phi k v}$
$= s_{111} = $ Poor

$Q_{121} = 0.1100/0.0533 + 0.3400/0.0800$ for s_{121}
$= $ Below Average

$Q_{131} = 0.3900/0.2267 + 0.3800/0.2133$ for s_{131}
$= $ Average

$Q_{141} = 0.3000/0.6133 + 0.3500/0.3600$ for s_{141}
$= $ Above Average

$Q_{151} = 0.2200/0.1067 + 0.1900/0.3067$ for s_{151}
$= $ Excellent

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For Arrow,

$s_{\phi v} = 1.0/1 + 1.0/2 + 1.0/3 + 1.0/4 + 1.0/5.$

In the case of 0.0 belief, the estimation of no (0.0) likelihood that the supplier's on-time delivery will rate Poor because no respondents in partition one scored this supplier as Poor. Since one respondent in partition two did rate the company's delivery performance as Poor, and this respondent's sales revenue volume was in the middle range, there is a 0.51 belief that there is a 0.04 probability that Arrow's on-time delivery will be Poor. The highest beliefs (0.39 and 0.38) are for low probabilities (0.2267 and 0.2133) that the supplier has Average on-time performance. The highest probabilities (0.6133 and 0.3600) for Above Average have among the highest beliefs (0.3 and 0.35, respectively). While one group of respondents considered a 0.3067 probability of occurrence with 0.19 belief, the other group held an even higher belief for a low probability (0.1067) of Excellence in on-time delivery by Arrow.

Beliefs ($\alpha_{\phi k j v}$) and corresponding probabilities ($a_{\phi k j v}$) are then defined as:

$\alpha_{1111} = 0.0000$ $\alpha_{1121} = 0.2500$
$a_{1111} = 0.0000$ $a_{1121} = 0.0400,$

$\alpha_{1211} = 0.1100$ $\alpha_{1221} = 0.3400$
$a_{1211} = 0.0533$ $a_{1221} = 0.0800,$

$\alpha_{1311} = 0.3900$ $\alpha_{1321} = 0.3800$
$a_{1311} = 0.2267$ $a_{1321} = 0.2133,$

$\alpha_{1411} = 0.3000$ $\alpha_{1421} = 0.3500$
$a_{1411} = 0.6133$ $a_{1421} = 0.3600,$

$\alpha_{1511} = 0.2200$ $\alpha_{1521} = 0.1900$
$a_{1511} = 0.1067$ $a_{1521} = 0.3067.$

According to Step 3 of the algorithm, all combinations $\varphi_{\phi v}$ of the five scores across both data partitions are considered for each outcome that sums to one, which in this example yields:

$\varphi_{\phi v 1} = a_{1111} + a_{1211} + a_{1311} + a_{1411} + a_{1511} = 1.0,$
$\varphi_{\phi v 2} = a_{1121} + a_{1221} + a_{1321} + a_{1421} + a_{1521} = 1.0.$

Possible n-tuples are (0.0000, 0.0533, 0.2267, 0.6133, 0.1067), and (0.0400, 0.0800, 0.2133, 0.3600, 0.3067). Following Step 4, a "weighted average" probability $b_{\phi v}$ for all $\varphi_{\phi v}$ is derived:

$b_{\phi v 1} = (0.0000)(1) + (0.0533)(2) + (0.2267)(3) + (0.6133)(4) + (0.1067)(5) = 3.7733,$

$b_{\phi v 2} = (0.0400)(1) + (0.0800)(2) + (0.2133)(3) + (0.3600)(4) + (0.3067)(5) = 3.8133.$

The minimum degree of belief in the on-time delivery then assessed according to Step 5 considers only the cases where belief is greater than zero:

$c_{\phi v 1} = $ Min {0.00, 1; 0.11, 1; 0.39, 1; 0.30, 1; 0.22, 1} $= 0.11,$
$c_{\phi v 2} = $ Min {0.25, 1; 0.34, 1; 0.38, 1; 0.35, 1; 0.19, 1} $= 0.19.$

Step 6 defuzzifies the expected score such that v_1's expected fuzzy score for On-time Delivery Performance is:

$$E(s_{11}) = [(0.1100)(3.7733) + (0.1900)(3.8133)] / [0.1100 + 0.1900] = 3.7987.$$

Applying the algorithm (with significant digit rounding) to the second supplier v_2 (Avnet) yields:

$Q_{112} = 0.2/0.0 + 0.4/0.0$
for s_{112} = Poor

$Q_{122} = 0.4/0.1 + 0.3/0.1$
for s_{122} = Below Average

$Q_{132} = 0.4/0.3 + 0.4/0.3$
for s_{132} = Average

$Q_{142} = 0.3/0.6 + 0.3/0.4$
for s_{142} = Above Average

$Q_{152} = 0.2/0.1 + 0.1/0.2$
for s_{142} = Excellent

Again, according to Step 3 of the algorithm, all combinations are considered for each outcome that sums to one, which in this example yields $a_{\phi kjv}$ combinations:

$\varphi_{\phi v 1} = a_{1111} + a_{1211} + a_{1311} + a_{1411} + a_{1511} = 1.0,$
$\varphi_{\phi v 2} = a_{1121} + a_{1221} + a_{1321} + a_{1421} + a_{1521} = 1.0,$

as for supplier v_1 (Arrow) but also:

$\varphi_{\phi v 3} = a_{1111} + a_{1221} + a_{1311} + a_{1411} + a_{1511} = 1.0,$
$\varphi_{\phi v 4} = a_{1111} + a_{1211} + a_{1321} + a_{1411} + a_{1511} = 1.0,$
$\varphi_{\phi v 5} = a_{1121} + a_{1221} + a_{1321} + a_{1421} + a_{1521} = 1.0,$
$\varphi_{\phi v 6} = a_{1121} + a_{1211} + a_{1321} + a_{1421} + a_{1521} = 1.0,$
$\varphi_{\phi v 7} = a_{1121} + a_{1221} + a_{1311} + a_{1421} + a_{1521} = 1.0.$

Continuing with steps 4 through 6 the defuzzified E(x) for Avnet, rounded to two decimal places is 3.69.

By step 7, the maximum and minimum scores over all suppliers were determined for each Delivery criterion attributes such that On-Time Delivery was recorded as 4.8250 and 2.5348, respectively. Thus, Arrow's score of 3.80 fits the qualitatively defined parameter of High (H), 0.79 of the maximum (3.8000/4.8250) and of Low (L) as the minimum relationship to Arrow's score is 0.67 (2.5348/3.800) with fuzzy membership then defined as .79/High + .67/Low . Similarly, Avnet is 0.76/High + 0.69/Low.

On-Time Delivery was then compared to Availability of Inventory, Shipping Accuracy, and Return Authorization; Availability of Inventory was compared to Shipping Accuracy, Return Authorization; and Shipping Accuracy was finally compared to Return Authorization to comprise all possible combination of I and J functions by Step 8. To allow a neutral first set of decision rules, it was determined to accept a supplier when belief and plausibility of the generated rules was 0.5; Steps 9 and 10 of the algorithm.

4.3. Results

The fuzzy probabilities from the respondents for the six suppliers are found by Steps 3 to 7 of the algorithm from four fuzzy probability (summation to 1.0) combinations for v_3; 8 combinations for v_4; 5 combinations for v_5 and 2 combinations for v_6. The results for the six suppliers in this example are as follows for our defined Delivery attributes: Table 3.

Next are the results for the six suppliers in this spreadsheet example for the Front Office Quality category: Table 4.

Lastly, the results for the six suppliers in this spreadsheet example are as follows for our defined Value-Added Services category: Table 5.

For illustrative purposes, the rule development is presented for the Delivery criterion having the attributes of On-Time Delivery, Availability of Inventory, Shipping

Accuracy and Return Authorization from Table 3 above. A pairwise comparison was then set up in a spreadsheet of these attributes for each supplier where, as stated, On-Time Delivery belief was on the set {High, Low} and Shipping Accuracy was on the set {Great, Small} based on the maximum and minimum values given in Table 6 below:

TABLE 3. SCORES FOR ALL SUPPLIERS ON ALL ATTRIBUTES OF DELIVERY CRITERION

Supplier	On-Time Delivery	Availability of Inventory	Shipping Accuracy	Return Authorization	Overall Average
v_1: **Arrow**	3.80	**3.77**	3.93	**3.80**	**3.82**
v_2: **Avnet**	3.69	3.63	3.90	3.68	3.72
v_3:**Future**	3.71	3.60	3.81	3.30	3.61
v_4: **Insight**	3.65	3.33	3.83	3.66	3.62
v_5: **Pioneer**	3.57	3.27	3.70	3.61	3.54
v_6: **TTI**	**3.89**	3.71	**4.06**	3.68	**3.83**

TABLE 4. SCORES FOR ALL SUPPLIERS FOR ALL ATTRIBUTES ON FRONT OFFICE QUALITY CRITERION

Supplier	Quote Completeness	Credit/PMT Terms	Non-Return Letters	Contract Terms	Overall Average
v_1: **Arrow**	**3.81**	**4.02**	3.60	**3.71**	**3.79**
v_2: **Avnet**	3.49	3.92	3.32	3.60	3.58
v_3:**Future**	3.53	3.81	3.34	3.42	3.53
v_4: **Insight**	3.55	3.68	3.36	3.51	3.53
v_5: **Pioneer**	3.68	3.74	3.51	3.41	3.58
v_6: **TTI**	3.79	3.81	**3.62**	3.54	3.69

TABLE 5. SCORES FOR ALL SUPPLIERS ON ALL ATTRIBUTES OF VALUE-ADDED SERVICES CRITERION

Supplier	Knowledge Specialists	Technical Design Help	e-Business Services	Sales/Mgt. Support	Customized Processes	Overall Average
v_1: **Arrow**	**3.90**	**3.51**	3.56	**3.80**	**3.60**	**3.67**
v_2: **Avnet**	3.64	3.46	3.41	3.57	3.34	3.48
v_3:**Future**	3.50	3.39	3.02	3.46	3.36	3.34
v_4: **Insight**	3.46	3.37	2.98	3.45	3.50	3.35
v_5: **Pioneer**	3.57	3.25	3.17	3.47	3.45	3.38
v_6: **TTI**	3.63	3.28	**4.19**	3.66	3.44	3.44

TABLE 6. COMPARISON RULE GENERATION FOR ON-TIME DELIVERY AND SHIPPING ACCURACY ATTRIBUTES OF DELIVERY CRITERION

Supplier	On-Time Delivery	Max	Min	High (H)	Low (L)	Shipping Accuracy	Max	Min	Great (G)	Small (S)	Select Supplier
Arrow	3.8000	4.8250	2.5348	0.79	0.67	3.93	4.86	2.87	0.81	0.73	0.50
Avnet	3.6900	4.8250	2.5348	0.76	0.69	3.90	4.86	2.87	0.80	0.74	0.50
Future	3.7100	4.8250	2.5348	0.77	0.68	3.81	4.86	2.87	0.78	0.75	0.50
Insight	3.6500	4.8250	2.5348	0.76	0.69	3.83	4.86	2.87	0.79	0.75	0.50
Pioneer	3.57	4.8250	2.5348	0.74	0.71	3.70	4.86	2.87	0.76	0.78	0.50
TTI	3.8900	4.8250	2.5348	0.81	0.65	**4.06**	4.86	2.87	0.83	0.71	0.50

Setting a level of acceptance of rules as $\Omega = 0.69$, then I functions for Delivery attributes On-time Delivery {H,L} and Shipping Accuracy {G,S} yield the following by Steps 9 and 10 of the algorithm where D_A is the decision to select the supplier.

$$I (H \subset D_A) = .50 \quad I (H \cap G \subset D_A) = .69$$
$$I (L \subset D_A) = .50 \quad I (H \cap S \subset D_A) = .63$$
$$I (G \subset D_A) = .50 \quad I (L \cap G \subset D_A) = .65$$
$$I (S \subset D_A) = .50 \quad I (L \cap S \subset D_A) = .63$$

With a threshold of $\Omega = 0.69$, the rules for selecting a supplier based on On-time Delivery and Shipping Accuracy are #1 and #2 below. In a similar manner, rules #3 through #9 are developed:

1. If On-Time delivery is High scored (4.825) and Shipping Accuracy is High scored (4.86) then supplier should be selected; certainty of belief = .74.
2. If On-Time delivery is High scored (4.825) and Shipping Accuracy is Low scored (2.87) then the Supplier should be selected; belief certainty = .71
3. If On-Time delivery is High scored (4.8250) and Availability of Inventory

is High scored (4.73), then the supplier should be selected; belief certainty = .69
4. If On-Time delivery is High scored (4.825) and Return Authorization is High scored (4.71) then supplier should be selected; certainty of belief = .70.
5. If Availability of inventory is High scored (4.73) and Shipping Accuracy is High scored (4.86) then the supplier should be selected; certainty of belief = .69.
6. If Availability of inventory is High scored (4.73) and Shipping Accuracy is Low scored (2.87) then the supplier should be selected; certainty of belief = .69.
7. If Availability of inventory is High scored (4.73) and Return Authorization is High scored (4.71) then the supplier should be selected; certainty of belief = .69.
8. If Shipping Accuracy is High scored (4.86) and Return Authorization is High scored (4.71), then the supplier should be selected; certainty of belief = 0.70.
9. If Shipping Accuracy is High scored (4.86) and Return Authorization is Low

scored (2.17), then the supplier should be selected; certainty of belief = 0.70.

The J functions yield no possible rules at a threshold of $\Omega = .70$ but all rules are plausible at a threshold of .50. At this threshold, however, all rules also become certain, so this threshold level fails to differentiate among the Delivery attributes when pair-wise compared.

V.   DISCUSSION

First, the example above shows only the Delivery attributes, but based upon the rules, there are three overriding attributes that affect supplier selection decisions when used in conjunction; on-time delivery, availability of inventory and return authorization. On-time delivery is always required to be highly scored. In particular, since shipping accuracy can be scored high or low, when considering the unions and intersections, this equates to a non-consideration. This is also true for the relationship of inventory availability to either high or low shipping accuracies. Thus, by rules #3, #4 and #7, the overriding rule becomes:

If on-time delivery is scored high, availability of inventory is scored high, and return authorization process is also scored high, then the supplier should be selected (minimum belief in the certainty of this rule is .69)

Shipping accuracy had the highest certainty when coupled with on-time delivery, but overall, it appears that the respondents believe that if the inventory is available and delivered on-time, then if the order is not accurate a good return authorization policy will suffice.

According to Table 3: TTI has the highest score (3.89) for On-Time delivery, while Arrow is highest for Availability of Inventory and Return Authorization. Thus, there is no definitive choice, although Arrow's competitiveness with TTI for On-Time Delivery could make it a leading contender. Still, Rule #1 has the greatest degree of

certainty of belief (.74) and given that TTI has both the highest On-Time Delivery score and the highest Shipping Accuracy score, this rule could definitely be applied, resulting in TTI being selected.

When the algorithm is repeated for the attributes related to Front-Office Quality the following rules would be considered:

1. If Quote Completeness & Turnaround is High scored (4.8171) and Credit & Payment Terms is High scored (4.79), then the supplier would be selected; certainty of belief = .72.
2. If Quote Completeness & Turnaround is High scored (4.8171) and Non-cancellable return letters is High scored (4.56), then the supplier would be selected; certainty of belief = .72.
3. If Quote Completeness & Turnaround is High scored (4.8171) and Contract Processing is High scored (4.59), then the supplier would be selected; certainty of belief = .72.
4. If Credit &Payment Terms is High scored (4.79) and Non-cancellable return letters is High scored (4.56), then the supplier would be selected; certainty of belief = .73.
5. If Credit & Payment Terms is High scored (4.79) and Contract Processing is High scored (4.59), then the supplier would be selected; certainty of belief = .74.
6. If Non-Cancellable Return Letters is High scored (4.56) and Contract Processing is High scored (4.59), then the supplier would be selected; certainty of belief = .73.

Of interest in these rules is that the certainty of belief for each rule in the Front Office Quality criterion has slightly stronger belief than do the rules for the Delivery

criterion. Thus, there is stronger consensus among the respondents regarding the attributes needed from the front office of the supplier. Basically, with at least belief of .72, all attributes within the Front Office Quality should be highly scored in order to select a supplier. From Table 4, no supplier was highest scored for each attribute. However, Arrow was the highest scored for all attributes except Non-cancellable Return Letters, where its score of 3.60 was slightly lower than that of TTI at 3.62.

Potentially, fewer consensuses led to fewer rules regarding Value-Added Services attributes. The following rules had certainty of belief greater than .70.

1. If Knowledge Specialists is High scored (4.8132) and Technical Design Services is High scored (4.47), then the supplier should be selected; certainty of belief = .72.
2. If Knowledge Specialists is High scored (4.8132) and Sales Management Support is High scored (4.78), then the supplier should be selected; certainty of belief = .72.
3. If Technical Design Services is High scored (4.47) and Sales Management Support is High scored (4.78), then the supplier should be selected; certainty of belief = .72.
4. If Customized Processing is High scored (4.47) and Sales Management Support is High scored (4.78), then the supplier should be selected; certainty of belief = .72.
5. If Technical Design Services is High scored (4.47) and Customized Processing is High scored (4.47), then the supplier should be selected; certainty of belief = .73.
6. If Knowledge Specialists is High scored (4.8132) and Customized Processing is

High scored (4.47), then the supplier should be selected; certainty of belief = .72.

Overall, E-business capabilities did not figure into the selection decisions according to these respondents for this group of suppliers. Of importance were high scores for Knowledge Specialists, Technical Design Services, Sales Management Support and Customized Processing. The last two rules, however, indicate that Customized Processing is a stronger belief factor in the selection decision than Knowledge Specialists. This could reflect that the purchasers are already sufficiently knowledgeable, but need a supplier that can translate their ideas into customized processes for implementation. From Table 5, Arrow is higher scored (3.60) than TTI (3.44) in Customized Processing. Arrow also scored higher in each of the other importance attributes. Although, TTI scored highest in E-business capabilities, this was not relevant overall to decision rules for supplier selection.

It should be noted that the maximum and minimum values are based on the mean score from the respondents for each supplier under each attribute. For example, under the criterion of Delivery, On-time delivery (Question 20 from the survey) as given in Table 23 had scores for Arrow, Avnet, Future, Insight, Pioneer and TTI of 3.80, 3.69, 3.71, 3.65, 3.57, and 3.89, respectively. With one standard deviation, these were 4.71, 4.56, 4.63, 4.62, 4.64 and 4.83, respectively while minus one sigma resulted in 2.88, 2.81, 2.79, 2.68, 2.53, and 2.96, respectively. Thus, when considering the maximum overall, all suppliers with scores within one standard deviation, 4.83 was used as the maximum for on-time delivery and 2.53 was set as the minimum. In a similar manner, the mean scores for availability of inventory (Question 21 on the survey) were determined for each supplier using the mean plus and minus one standard deviation then selecting the overall maximum and minimum values; 4.73

and 2.39, respectively. For shipping accuracy (Question 22) the maximum and minimum scores based on one standard deviation were 4.86 and 2.87, respectively, and for Return Authorization these were 4.71 and 2.17, respectively.

The use of one standard deviation can be statistically related to these belief measures. Shipley and deKorvin (1995) showed that the setting of the maximum and minimum values should be done carefully since the degree of certainty will increase or decrease based on these values. While using $\pm 3\sigma$ provides more statistical confidence in the certainty of belief in any rule, the belief values themselves will be low. This statement is similarly relevant for $\pm 2\sigma$, so by using $\pm 1\sigma$, the statistical confidence in the rules is approximately the same as the certainty of belief in the rules.

VI. CONCLUSION

The strength of this process is that it can be performed using a spreadsheet and the rule-based results can be interpreted based on setting of values for the maximum and minimums by which the scores will be contained or intersect resulting in the I and J functions. While the J functions yielded every combination of pair-wise relationships to be plausible at level of possibility .5, the I functions were more discriminatory for certainty of belief in the rules. Thus, setting the maximum and minimum values is crucial to achieving certainty and possibility levels.

For the application, the research of Shipley and deKorvin (1995) was used allowing the maximum and minimum values to be within one standard deviation from the mean. While this process does not have to be followed by the decision maker in determining the relevant rules, it does afford a degree of statistical confidence to be attributed to the generated rules. Obviously, setting larger ranges based on two or even three standard deviations affords greater confidence in the results, but by the algorithmic process it can be easily observed that the certainty of belief and the possibility beliefs for all generated rules will be lowered. Thus, higher confidence results in lower belief in the generated rules. It was believed that the one standard deviation approach provided sufficient confidence in the beliefs for the I (and J) functions. Therefore, the approach detailed herein provides a reasonable approach for setting the maximum and minimum values that anchor the decision rules.

In order then to validate the attributes used in the survey as realistic to actual criteria used for supplier selection, the work of Simpson, Siguaw and White (2002) was used. The results of their study confirmed not just what attributes were considered in selecting suppliers but also what percent of those respondents used an attribute on an official evaluation form. Therefore, by tying it to the supplier survey instrument that was developed and administered by an author of this work, an attempt has been made to validate the survey instrument's questions which the industry experts addressed. A drawback can be the interpretation of the attributes used in this application to those stated in the Simpson, Siguaw, and White (2002) work for while some statements were worded exactly, others required interpretation. Based upon the attributes used in this study to aspects from the forms used to officially rate a supplier, however, the percentage of parallelism ranged from 31% (availability of inventory) to 61% (on-time delivery) for attributes under the Delivery category; 10.7% (credit & payment terms) to 98.8% (contract processing) under Front-Office Quality; and 32.1 % (knowledge and technical design services) to 85.7% (e-business) for Value-Added Services. This is an important result because it reveals and captures the emphasis placed by supply chain managers on multiple performance criteria of suppliers. Indeed, the credit and payment terms seemed an anomaly since most attributes represented a

mapping to forms percentages of 31% or higher. Thus, the survey questions are substantiated as being relevant to the supplier selection decision making process as are the criteria that consolidate the attributes used in selection.

This fuzzy set, rule-based approach to supplier selection decision making is a unique contribution to the existing literature. A benefit is that it includes attributes that have been shown to be real-world considerations beyond the traditional price, delivery and quality. While delivery and quality are considered in the criteria, these aspects were further refined to more specific attributes. Thus, the contribution of the application is presentation of a rather rigorous evaluation of the survey data described in section IV. While it is a strength of this research that survey data can be utilized in the algorithmic process, a limitation is that the belief measures were not directly queried of the respondents.

The benefit of the fuzzy rule-based approach beyond the pairwise comparison approaches in either the crisp or fuzzy environs is that the results are more defined than a score or a comparison score by which a rank order can be determined. Decision makers using the algorithmic approach developed in this work for supplier selection are given scenarios by which the relationship of attributes combines toward the selection process. A supplier's degree of membership based on a score that fits a pre-determined qualitatively-defined parameter allows certainty of belief to be assessed. Such certainty of belief a priori to the action of selection of a supplier guides the decision maker who may be more risk averse since setting of the limits on the level of certainty for acceptance of a functional rule is at the discretion of the decision maker. The model presented herein is also applicable without loss of generality to other criteria and other attributes. The process is responsive to setting of different maximum and minimum values for defined qualitative variables beyond those given as High, Great, Small, etc. Thus, the model allows flexibility which includes allowing the decision maker to select the value of belief (Ω) that would be acceptable; thereby either expanding the number of rules or restricting the number of rules. The process is, therefore, customizable to the decision maker while providing rules for corporate guidelines in supplier selection. Overall, the fuzzy rule-based model developed and illustrated herein contributes a dynamic supplier selection process as an alternative to the more frequently utilized checkbox or scoring model approaches.

VII.   REFERENCES

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