Allowing Promotion, Deterioration, Time Value of Money and Shortages in Economic Production Quantity (EPQ) Model for Price Dependent Declined Demand

Monalisha Pattnaik*
Utkal University, Bhubaneswar, Odisha, India

A deteriorating inventory model using time-value of money with price dependent declined quadratic demand allowing promotion policy is developed for a deterministic inventory system. This study applied the discounted cash flows (DCF) approach for problem analysis with shortages in economic production quantity (EPQ) model. The objective of this model is to maximize the net present value profit so as to determine the optimal time period, promotion factor and order quantity. The numerical analysis shows that an appropriate policy can benefit the retailer and promotional policy is important, especially for deteriorating items. Finally, sensitivity analysis of the optimal solution with respect to the major parameters are also studied to draw some decisions with managerial implications for competitive advantage.

* Corresponding Author. Email address: monalisha_1977@yahoo.com

I. INTRODUCTION

Inventory modelling is an important part of operations research, which may be used in solving variety of warehousing and storing problems. It plays a significant role in production and operations function of supply chain management to make it applicable and flexible in real life situation. Researchers are engaged in modifying existing models on different parameters economy and have been in excess of 22% of the nation's gross national product over the past few decades. As millions of dollars are tied up in inventories, proper management of these inventories can prove to be very profitable in a manufacturing system. A major concern of inventory management is to know when and how much to order or manufacture so that the total cost per unit time will be minimized. The constituents of total inventory cost is carrying cost, shortage cost, replenishment cost or ordering cost and the purchase cost or production cost but the time value of money is not considered explicitly in analysing inventory systems, although the cost of capital tied up in inventories and it is included in the carrying cost. So inventory management plays a significant role for production system in business since it can help companies to reach the goal of ensuring prompt delivery, avoiding shortages, helping sales at competitive prices and so forth for achieving competitive advantage in the globe.

Furthermore, retailer promotional activity has become more and more common in today's business world. For example, Wal-Mart and Costco often try to stimulate demand for specific types of electric equipment by offering price discounts; clothiers Baleno and NET make shelf space for specific clothes items available for longer periods; McDonald's and Burger King often use coupons to attract consumers. Other promotional strategies include free goods, advertising, and displays and so on. The promotion policy is very important for the retailer. How much promotional effort the retailer makes has a big impact on annual profit. Residual costs may be incurred by too many promotions while too few may result in lower sales revenue. Tsao and Sheen (2008) discussed dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment. Salameh et al. (1999) studied an EOQ inventory model in which it assumes that the percentage of on-hand inventory wasted due to deterioration is a key feature of the inventory conditions which govern the item stocked. The effect of deteriorating items on the instantaneous profit maximization replenishment model under promotion is considered in this model. The market demand may increase with the promotion of the product over time when the units lost due to deterioration. In the existing literature about promotion it is assumed that the promotional effort cost is a function of promotion. Tripathy et al. (2012) investigated an optimal EOQ model for deteriorating items with promotional effort cost. Evidence can be found in the increasingly prosperous revenue and yield management practices and the continuous shift away from supply-side cost control to demand-side revenue stimulus. This model introduces a modified economic order quantity model in which it assumes that a percentage of the on-hand inventory is wasted due to deterioration. There is hidden cost not account for when modelling inventory cost. This model studies the problem of promotion for a deteriorating item subject to loss of these deteriorated units. This model postulates that measuring the behaviour of production systems may be achievable by incorporating the idea of retailer in making optimum decision on replenishment with wasting the percentage of on-hand inventory due to deterioration and then compares the optimal results with none wasting the percentage of on-hand inventory due to deterioration traditional model. This model addresses the problem by proposing an inventory model under promotion by assuming that the units lost due to deterioration of the items. In this model, promotional effort and replenishment decision are adjusted arbitrarily upward or downward for profit maximization model in response to the change in market demand within the planning horizon. The objective of this model is to determine the optimal time length, optimal units lost due to deterioration, the promotional effort and the replenishment quantity with variable ordering cost so that the net profit is maximized in an instantaneous replenishment fuzzy economic order quantity model and the numerical analysis show that an appropriate promotion policy can benefit the retailer and that promotion policy is important in fuzzy space, especially for deteriorating items. Finally, sensitivity analyses of the optimal solution with respect to the major parameters are also studied to draw the managerial insights.

The classical analysis builds a model of an inventory system and it calculates the economic order quantity which minimizes the total inventory cost satisfying the optimization criterion. One of the unrealistic assumption is that items stocked preserve their physical characteristics during their stay in inventory for long run. Items in stock are subject to many possible risks, e.g. damage, spoilage, dryness, vaporization etc., those results decrease of usefulness of the original one and a cost is incurred to account for such risks of the product. The problem of deteriorating inventory has received considerable attention in recent years. This is a realistic trend since most of the products such as medicine, dairy products and chemicals starts to deteriorate once they are produced.

Ghare and Schrader (1963) were among the first authors who studied inventory problems considering deterioration of items. Since then, a number of studies on deteriorating items have been done, Covert and Philip (1973) assumed two parameter Weibull distribution deterioration to consider varying deterioration rate of the product. Dave and Patel (1981) incorporated the effect of deterioration and time-varying demand to derive an economic order quantity (EOQ) with equal replenishment cycles. Hollier and Mak (1983) and Bahari-Kashani (1989) extended the model by Dave and Patel (1981) to consider variable replenishment periods. Sachan (1984) and Wee (2001, 1997) extended the model by Dave and Patel (1981) to allow for shortages. Wee (1995) later developed a replenishment policy for items with a price-dependent demand and a varying rate of deterioration.

Most researches in inventory do not consider the time-value of money. This is unrealistic, since the resource of an enterprise depends on when it is used and this is highly correlated to the return of investment. Therefore, the time-value of money should be taking into account especially when investment and forecasting are considered. Buzacott was the first author to include the concept of inflation in inventory modelling. He developed a minimum cost model for a single item inventory with inflation. Mishra (1979) simultaneously considered both the inflation and the time-value of money for internal as well as external inflation rate and analyzed the influence of interest rate and inflection rate on replenishment strategy. Chandra and Bahner (1985) extended the result in Mishra (1979) to allow for shortages. Sarkar and Pan (1994) assumed a finite replenishment model and analysed the effects of inflation and time-value of money on order quantity when shortages and allowed. Wee (2001) developed a replenishment policy for items with a price-dependent demand and a varying rate of deterioration. Hariga (1995) extended the study to analyze the effect in inflation and time-value of money of an inventory model with time-dependant demand rate and shortages. Bose et al. (1993) developed an EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting. Abad (1996), Chang et al. (1999), Goyal et al. (2001) and Chung et al. (1993) investigated deterministic deteriorated economic order quantity models. Pattnaik (2011), (2012) and (2013) derived different types of deterministic inventory models for deteriorating items in finite horizon. The present deteriorating inventory model using Weibull distribution and time-value of money with price dependent declined quadratic demand allowing promotion policy is developed for a deterministic inventory system. This study applied the discounted cash flows (DCF) approach for problem analysis with shortages in economic production quantity (EPQ) model.

The objective of this paper is to maximize the net present-value profit so as to determine the optimal period and order quantity by using time-value of money and price dependent declined quadratic demand allowing shortages with promotional activities. Replenishment decision during planning horizon due to deterioration for maximizing the net present value profit in response to change the market demand and is also considered. The major assumptions used in the above research article summarized in Table1. The remainder of paper organized in section 2 is assumptions and notations for development of the model. The mathematical model is developed in section 3. Optimization is given in section 4. Numerical example is presented to illustrate the development of model in section 5. The sensitivity analysis carried out in section 6 to observe the changes in optimal solution. Finally section 7 deals with conclusion and concluding remarks.

TABLE 1. SUMMARY OF RELATED RESEARCHES

Authors Published Year	Model Structure	Promotion	Demand	Demand Pattern	Deterioration	Allowing shortage	Time value of Money	Planning	New form
Wee et al. 1993	EPQ, Profit	No	Price	Linear Decrease	Yes Weibull	Yes	Yes	Finite	Product rate
Wee et al. 2001	EOQ, Profit	No	Price	Declined Linear	Yes Weibull	Yes	Yes	Finite	DCF approach
Pattnaik 2013	EOQ, Profit	No	Constant Deterioration	Constant	Yes Constant Wasting	No	No	Finite	Unit lost due to deterioration and various ordering cost
Tsao et al. (2008)	EOQ, Profit	Yes	Time and Price	Linear and decreasing	Yes	No	No	Finite	Promotion
Datta etal. 1991	EOQ, Profit	No	Time	Linear Decrease	No	Yes	Yes	Finite	Inflation
Hariga 1995	EOQ, Profit	No	Time	Decrease	No	Yes	Yes	Finite	Inflation
Present Paper 2015	EPQ, Profit	Yes	Price	Declined Quadratic	Yes Weibull	Yes	Yes	Finite	DCF approach

II. ASSUMPTION AND NOTATION

- The distribution of time until deterioration the item follows a two-parameter Weibull distribution.
- Deterioration occurs as soon as the items received into inventory.
- There is no replacement or repair of deteriorating items during the period under consideration.
- The demand rate is a decreasing quadratic function of selling price.
- The replenishment rate is instantaneous; the order quantity and the replenishment cycle is same for each period.
- The system operates for a prescribed period of a planning horizon.
- Shortages are completely back ordered.
- The order quantity, inventory level, replenishment epoch and demand are treated as continuous variables while the number of replenishments is restricted to an integer variable.
- Continuous cost compounding is implemented throughout the analysis.
- Product transactions are followed by instantaneous cash flow.

s Per unit selling price of the items (s/unit) where $c < s < \frac{l}{m}$, since cost price must be less than the selling price for a profit model.

d(s) Demand rate, $d(s) = l - ms - s^2$, where, $l > 0 \; and \; m > 0$

C Per unit cost of items (s/unit)

T_1 Replenishment time for positive inventory

$T - T_1$ Replenishment time for shortages

$I_2(t_2)$ Inventory level at any time, $0 < t_2 < T - T_1$ (Negative inventory)

H Planning horizon

T Replenishment cycle

N Number of replenishment during planning horizon; $N = \frac{H}{T}$

R Interest rate

Q The 2nd, 3rd, ... Nth replenishment size (units)

I_m maximum inventory level

c_1 Cost per replenishment when t= 0 ($)

c_2 Per unit holding cost per unit time (s unit / unit time)

c_3 Per unit shortage cost per unit time (s / unit / unit time)

ρ The promotional effort factor per cycle,

PE(ρ) The promotional effort cost, PE(ρ)= $K_1(\rho-1)^2 d(s)^{\alpha_1}$ where, $K_1>0$ and α_1 is a constant,

One distribution that has been used extensively in literature to pattern a varying rate of deterioration is the Weibull distribution. The two parameter Weibull density function is

$$f(t) = \alpha\beta t^{\beta-1}.\, e^{-\alpha t^{\beta}} \tag{1}$$

Here t is the time to deterioration, $t>0$, $f(t)$ probability density function α the scale parameter, $t>0$ and β the shape parameter, $\beta > 0$ .This probability density function represent the hand inventory deterioration that may have an increasing, decreasing or constant rate depending on the value of β. When $\beta > 1$, deteriorating rate increases with time e.g. Fish and vegetables. When $\beta < 1$ deteriorating rate decreases with time e.g.: light bulb where the initial deterioration breakdown rate may be higher due to irregular voltages and handling. When $\beta = 1$ deterioration rate is constant; e.g. electronic products. Here, the two-parameter Weibull distribution is reduced to an exponential distribution. The instantaneous rate of deterioration of the on hand inventory is given by $\alpha\beta t^{\beta-1}$.

III. MATHEMATICAL MODEL

The replenishment lot size of I_m is replenished initially at $t=0$. During the period T_1, the inventory level decreases due to demand an deterioration until it is zero at $t = T_1$. During the time interval, $T_2 (T_2 = T - T_1)$, shortages occurred are accumulated until $t = T$ before they are backordered. The inventory system at any time t can therefore be represented by the following equations.

$$\frac{dI_1(t_1)}{dt_1} + \alpha\beta t_1^{\beta-1} I_1(t_1) = -d(s),\ 0 \le t_1 \le T_1 \tag{2}$$

$$\frac{dI_2(t_2)}{dt_2} = -d(s),\ 0 \le t_2 \le T - T_1 \tag{3}$$

The first-order differential equation can be solved by using the boundary conditions

$$I_1(0) = I_m,\ I_2(0) = 0,$$

$$I_1(t_1) = \frac{I_m - d(s)\int_0^{t_1} e^{\alpha u^{\beta}} du}{e^{\alpha t_1^{\beta}}};\quad 0 \le t_1 \le T_1 \tag{4}$$

$$I_2(t_2) = -d(s)t_2;\quad 0 \le t_2 \le T - T_1 \tag{5}$$

Since $I_1(T_1) = 0$, one can derive from (4) the maximum inventory level as

$$I_m = d(s)\int_0^{T_1} -e^{\alpha u^{\beta}} du = d(s)\int_0^{T_1} \sum_{n=0}^{\infty} \frac{\alpha^n u^{n\beta}}{n!} du\alpha = d(s)\sum_{n=0}^{\infty} \frac{\alpha^n T_1^{n\beta+1}}{n!(n\beta+1)} \tag{6}$$

Assuming a very small α value $(\alpha \le 0.05)$ the approximate solution is found by neglecting the second and higher – order terms of α, one has

$$\mathrm{Im} \approx d(s)\left(T_1 + \frac{\alpha T_1^{\beta+1}}{\beta+1} \right) \tag{7}$$

The total cost in this model includes the replenishment cost, material cost, holding cost and shortage cost. The objective is to maximize the total profit when the time-value of money with compounding interest rate is considered. The detailed analysis of each cost function is given below.

Present Value Sales Profit

During the period T_1, the replenished inventory is being consumed due to demand and deterioration. At $t=T_1$, all the shortages during the period $T-T_1$, are backordered with an instantaneous cash transactions during sales, the present-value sale revenue is

$$R = s\int_0^{T_1} d(s)e^{-rt_1}\,dt_1 + se^{-rT}\int_0^{T-T_1} d(s)\;dt_2$$

$$= s\,d(s)\left(\frac{1-e^{-rT_1}}{r} + e^{-rt}\left(T - T_1\right) \right) \tag{8}$$

Assuming a very small r value $\left(r \le 0.08\right)$ approximate solutions can be found by neglecting the second and higher-order terms of

$$r.\,R \approx s\,d(s)\left(T - \frac{rT_1^2}{2} - rT^2 + rT_1T \right) \tag{9}$$

Present Value Ordering Cost

Since replenishment is each cycle is done at the start of each cycle, the present-value replenishment cost is

$$C_0 = c_1 \tag{10}$$

Present Value Inventory Cost

Inventory occurs during period T_1, therefore, the present value inventory cost during the period is

$$C_H = c_2\int_0^{T_1} I_1(t_1)e^{-rt_1}dt_1$$

$$= c_2\int_0^{T_1}\left\{ d(s)\frac{\int_0^{T_1} e^{\alpha u^\beta}du - \int_0^{t_1} e^{\alpha u^\beta}du}{e^{\alpha t_1^\beta}} \right\} e^{-rt_1}dt_1$$

$$= c_2\int_0^{T_1}\left\{ d(s)\sum_{n=0}^{\infty}\frac{\alpha^n\left(T_1^{n\beta+1} - t_1^{n\beta+1}\right)}{n!\left(n\beta+1\right)} \times \left[\sum_{n=0}^{\infty}\frac{\left(-\alpha t_1^\beta\right)^n}{n!}\right]\right\}\left[\sum_{n=0}^{\infty}\frac{\left(-rt_1\right)^n}{n!}\right]dt_1$$

Assuming very small α and r value (see above) the approximate solution is found by neglecting second and higher-order terms of α and r and terms containing αr. Consequently,

$$C_H \approx C_2 d(s)\left[\frac{T_1^2}{2} - \frac{rT_1^3}{6} + \frac{\alpha\beta T_1^{\beta+2}}{(\beta+1)(\beta+2)} \right] \tag{11}$$

Present ValueShortage Cost

The maximum shortage level $I_b = d(s)(T - T_1)$. All shortage during $(T - T_1)$ be completely backordered at T. The present value shortage cost for the period is

$$C_S = c_3\int_0^{T-T_1}\left[-I_2(t_2)\right]e^{-r(T_1+t_2)}dt_2$$

$$= c_3\int_0^{T-T_1}\left[d(s)(t_2)\right]e^{-r(T_1+t_2)}dt_2$$

$$= \frac{c_3 d(s)}{r^2}\left[e^{-rT}\left(-rT + 1 + rT_1\right) + e^{-rT_1}\right]$$

Assuminga very small r value (see above) approximate solutions are founded by neglecting second and higher order terms of r, on has

$$C_S \approx \frac{C_3 d(s)}{6}\left(3T^2 - 6TT_1 - 2rT^3 + 3rT^2T_1 + 3T_1^2 - rT_1^3\right) \tag{12}$$

Present Value Item Cost

Replenishment is done at $t=0$ and T; the replenishment items are consumed by demand as deterioration during T_1 . The present-value cost, C_P , therefore includes item cost and deterioration cost, one has

$$C_P = cI_m + ce^{-rT}\int_0^{T-T_1} d(s)dt_2 \tag{13}$$

Assuming very small α and r values (see above) the approximate solution is found by neglecting the second and higher order terms of α and r and the terms containing $\alpha\, r$. Consequently,

$$C_P \approx cd(s) + \left(T - rT^2 + rT_1T + \frac{\alpha T_1^{\beta+1}}{\beta+1}\right) \tag{14}$$

Promotional Effort Cost

Promotional effort cost is a function of promotional effort factor and selling price, defined as

$$C_{PE} = K_1(\rho - 1)^2\{d(s)\}^{\alpha_1} \tag{15}$$

Present Value Net Profit

The first cycle present-value net profit is

$$R - C_O - C_H - C_S - C_P - C_{PE} = \pi_1 \tag{16}$$

There are N cycles during the planning horizon. Some inventory is assumed to start and end at zero, an extra replenishment at $T=H$ is required to satisfy the back orders of the last cycle in the planning horizon. Therefore, the total number of replenishment = $N+1$ times; the first replenishment lot are = I_m and the 2nd, 3rd,Nth replenishment lot size

$$l = I_m + \int_0^{T-T_1} d(s)\,dt_2 \tag{17}$$

and the last or $(N+1)$th replenishment lot size

$$\int_0^{T-T_1} d(s)dt_2 \tag{18}$$

The time-value of money affects all the replenishment periods and therefore must be considered separately, the total net present-value profit for the planning horizon is

$$\begin{aligned}\pi(t, T_1, N) &= \pi_1\left(1 + e^{-rT} + e^{-2rT} + e^{-3rT} + \ldots.. + e^{-(N-1)rT} - c_1e^{-rH}\right)\\ &= \sum_{n=0}^{N-1}\pi_1 e^{-rnT} - c_1e^{-rH} = \pi_1\left(\frac{1-e^{-rnT}}{1-e^{-rT}}\right) - c_1e^{-rH}\end{aligned} \tag{19}$$

where, $T=H/N$ and π_1 is derived by substituting (8) to (15) into (16).The optimization problem of this study can be formulated by maximizing (19) subject to $c < s < \frac{l}{m}$ and $0 < T_1 < T$.

IV. MODEL ANALYSIS

The following heuristic technique is derived the optimal s, T_1 and N values;

Step1:

Start by choosing a discrete variable N, where N is any integer number equal or greater than 1;

Step2:
Take the partial derivatives of $\pi(s, T, N)$ with respect to T_1 and s, and equate the results to zero, the necessary conditions for optimality are $\frac{\partial}{\partial T_1}\pi(s,T,N)=0$ and $\frac{\partial}{\partial s}\pi(s,T_1,N)=0$

Step3:
For different integer N values, derive T_1^* and s^* from above two equations substitute (s^*, T_1^*, N) into (19) to derive (π^*, T_1^*, N)

Step4:
Repeat step 2 and 3 for all possible N values within the lower and upper bound until the maximum $\pi(s^*, T_1^*, N)$ is found. The (s^*, T_1^*, N^*) and $\pi(s^*, T_1^*, N)$ values constitute the optimal solution and they satisfy the following conditions:

$$\Delta\pi(s^*, T_1^*, N^*) < 0 < \Delta\pi(s^*, T_1^*, N^* - 1) \tag{20}$$

Where
$\Delta\pi(s^*, T_1^*, N^*) = \pi(s^*, T_1^*, N^*+1) - \pi(s^*, T_1^*, N^*)$
substitute (s^*, T_1^*, N^*) into (17) to derive the 2nd, 3rd, ..., Nth replenishment lot size. If the objective function is concave, the following sufficient condition must be satisfied;

$$\left(\frac{\partial \pi}{\partial s \partial T_1}\right)^2 - \left(\frac{\partial^2 \pi}{\partial T_1^2}\right)\left(\frac{\partial^2 \pi}{\partial s^2}\right) < 0 \tag{21}$$

and any one of the following

$$\left(\frac{\partial^2 \pi}{\partial T_1^2}\right) < 0, \left(\frac{\partial^2 \pi}{\partial s^2}\right) < 0 \tag{22}$$

Since the total net present-value profit for the planning horizon π is a very complicated function due to high-power expression of the exponential function, it is not possible to show analytically the validity of the above sufficient conditions, a search procedure is used instead. The computation al results are show in the following illustrative example.

TABLE 2. OPTIMAL VALUES OF THE PROPOSED MODEL

Model	Itera tion	N	s	T_1	T	$d(s)$	Q	ρ	PE	π_1	Profit $=\pi$
Quadratic $d(s)$	63	3.8844 18	9.9010 56	1.4111 43	2.5743 88	62.364 86	523.61 47	3.261 07	637.67 28	1121.7 18	4341.2 21
Quadratic $d(s)$	42	7.1309 84	9.1004 40	0.7975 516	1.4023 31	80.780 24	113.29 87	-	-	304.76 26	2157.2 57
% change	-	83.58	-8.09	-43.48	-45.53	29.53	-78.36	-	-	-72.83	-50.31

V. NUMERICAL EXAMPLES

Optimal replenishment and pricing policies for the maximum present-value profit may be derived by using the methodology given in the preceding sections; this will help managers to improve their replenishment and pricing decisions. The replenishment cost, c_1 is \$80/order, the annual inventory cost c_2 is \$0.6/unit/year, the annual shortage cost c_3 is \$1.4/unit/year. The unit item cost c is \$5/unit, the scale and the shape parameters of the deterioration rate are α =0.05 and β =1.5 respectively. The annual interest rate, r is 0.08, the yearly demand rate $d(s)$ is $200\text{-}4s\text{-}s^2$ unit/year and the planning horizon, H is 10 years.

For the given data the total net value profit for the planning horizon π is Rs.4341.221, the number of replenishment during planning horizon N, is 3.884418, per unit selling price of the items is Rs. 9.901056, time with positive inventory T_1, is 1.411143, replenishment cycle T is 2.574388, selling price dependent declining quadratic demand $d(s)$ is 62.36486 and the replenishment size Q is 523.6147, the promotional effort factor ρ is 3.26107 and the promotional effort cost PE is 637.6728. The total number of order is therefore N+1=5. All the decision parameters are compared with the other model related to the declined quadratic demand $d(s)$ which is also related to the selling price and not allowing the promotional activities. It is observed that the number of replenishment during planning horizon, N and demand rate $d(s)$, are less than that the compared model. But other parameters like s, $T_1, T, Q, \pi_1 and\, \pi$ are more than that of the compared model which gives the better result in the present model for managerial decision making process.Fig.1 represents 2-dimensional plot of unit selling price s and demand rate $d(s) = 200 - 4s - s^2$. Similarly Fig. 2 depicts the mesh plot of T, T_1 and total present value profit π .

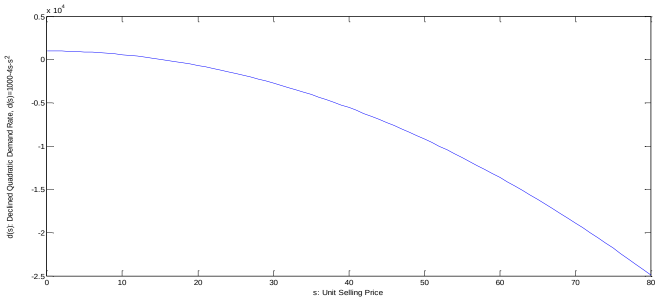

FIGURE 1. 2-DIMENSIONAL PLOT OF UNIT SELLING PRICES AND DEMAND RATE $\mathbf{d(s) = 200 - 4s - s^2}$

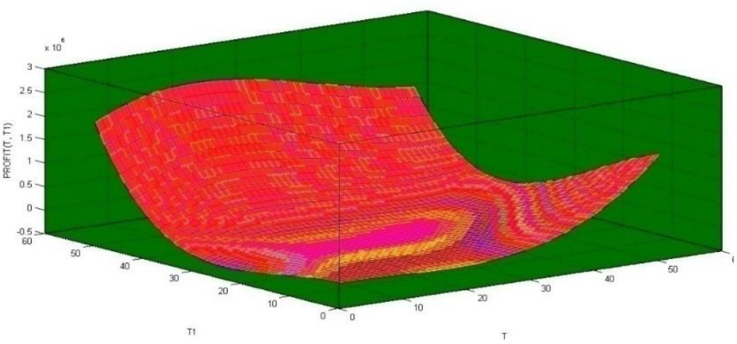

FIGURE 2. 3-DIMENSIONAL MESH PLOT OF T, T_1 AND NET PROFIT π

TABLE 3. SENSITIVE ANALYSIS OF THE PARAMETERS $\beta, r, H, c_1, c_{2,} c_3, c, \alpha, K_1 AND\ \alpha_1$

parameter	Value	Iteration	N	S	T_1	T	$d(s)$	Q	ρ	PE	π_1	Profit = π	Change in Profit
β	1.3	60	3.8414	9.9076	1.4369	2.6032	62.36382	531.2160	3.2797	646.6210	1133.896	4339.748	-0.0339
	1.7	61	3.9248	9.8950	1.3875	2.5479	62.4376	516.6471	3.2437	629.3972	1110.420	4342.197	0.0225
	1.9	63	3.9627	9.8894	1.3658	2.5236	62.20906	510.2645	3.2277	621.7670	1099.975	4342.820	0.0368
r	0.06	88	3.3606	9.9684	1.6381	2.9756	62.50898	659.1197	3.6455	850.4360	1413.367	4717.826	8.6751
	0.07	90	3.6199	9.9336	1.5176	2.7625	62.64217	585.4203	3.4404	733.6288	1254.855	4518.393	4.0812
	0.09	90	4.1528	9.8708	1.3170	2.4080	60.7574	471.5224	3.1036	558.3414	1009.176	4182.864	-3.6478
H	8	73	3.1075	9.9011	1.4111	2.5744	61.58919	523.6147	3.2611	637.6728	1121.718	3456.977	-20.3686
	11	71	4.2729	9.9011	1.4111	2.5744	63.08411	523.6147	3.2611	637.6728	1121.718	4783.344	10.1843
	12	71	4.6613	9.9011	1.4111	2.5744	62.36382	523.6147	3.2611	637.6728	1121.718	5225.466	20.3686
c1	75	68	3.9079	9.8980	1.4035	2.5589	62.36382	519.4719	3.2510	632.7645	1119.962	4361.702	0.4718
	85	95	6.9122	9.1071	0.8211	1.4467	80.63233	116.6690	1.0000	0.0000	309.3292	2121.157	-51.1392
	90	98	3.8394	9.9070	1.4260	2.6046	62.22335	531.7113	3.2805	647.2177	1124.825	4300.605	-0.9356
c2	0.5	65	3.778	9.9038	1.4947	2.6472	62.29955	548.8433	3.3277	675.1249	1175.192	4423.451	1.8942
	0.8	61	4.0829	9.8953	1.2680	2.4492	62.50184	481.7134	3.1465	575.9571	1032.598	4200.006	-3.2529
	0.9	79	4.1752	9.8925	1.2062	2.3951	62.56844	464.1686	3.0970	550.3100	995.1542	4138.948	-4.6594
c3	1.2	67	3.7345	9.8998	1.3915	2.6778	62.39476	559.1476	3.3463	687.0010	1192.606	4437.823	2.2252
	1.5	112	3.9538	9.9013	1.4194	2.5292	62.35906	508.4824	3.2236	616.6820	1091.345	4298.963	-0.9734
	1.7	81	4.0826	9.9010	1.4334	2.4494	62.3662	482.3503	3.1572	580.4631	1038.617	4224.220	-2.6951
c	4	80	3.4040	9.6520	1.6884	2.9378	68.2309	793.1539	3.9566	1192.917	1919.119	6519.119	50.1679
	6	64	4.4072	10.1662	1.1877	2.2690	55.98358	341.8833	2.6911	320.2245	618.9323	2711.776	-37.5343
	8	83	4.9701	10.8122	0.9520	2.0118	39.84753	154.1979	1.9233	67.9410	135.1067	655.5698	-84.8990
α	0.04	64	3.8034	9.9063	1.4698	2.6292	62.24002	541.1542	3.3068	662,3824	1156.672	4383.258	0.9683
	0.06	63	3.9592	9.8963	1.3591	2.5258	62.47805	508.2453	3.2203	615.9792	1090.849	4302.872	-0.8834
	0.08	66	4.0933	9.8881	1.2703	2.4430	62.67308	482.4716	3.1502	579.5093	1038.547	4235.118	-2.4441
K_1	3	80	4.2862	9.7473	1.2882	2.3331	66.00094	363.9149	2.3631	367.8903	827.680	3531.596	-18.6497
	4	85	4.6496	9.6308	1.1942	2.1507	68.72449	286.8056	1.9402	243.0160	679.9465	3145.509	27.5432
	5	93	4.9605	9.5436	1.1241	2.0159	70.7453	242.9597	1.7034	175.0144	592.6382	2923.806	-32.6501
α_1	1.1	68	4.2421	9.8134	1.3020	2.3573	64.44358	361.5605	2.3798	372.2013	831.6903	3512.150	-19.0976
	1.2	77	4.7594	9.6740	1.1703	2.1011	67.71772	256.8913	1.8053	204.0812	630.9191	2986.804	-31.1990
	1.5	68	6.3269	9.2940	0.8947	1.5806	76.44556	139.8804	1.1575	33.1767	374.3637	2352.564	-45.8087

VI. SENSITIVE ANALYSIS

It is interesting to investigate the influence of major parameters $\beta, r, H, c_1, c_2, c_3, c, \alpha, K_1, \alpha_1$.

- N , s , and *d(s) and* π are insensitive to the parameter β . T_1, T, Q, ρ, PE and π_1 are moderately sensitive to the parameter β .
- N, s, T_1,T, Q, ρ and *d(s)*are moderately sensitive to parameter r but $PE, \pi_1 and\, \pi$ are sensitive to the parameter r.
- N,T_1 and T are moderately sensitive to parameter H but s , *d(s)*and Q are insensitive to the parameter H and π is sensitive to the parameter H.
- N,Q, ρ, PE, π_1, π and *d(s)* are sensitive to the parameter c_1 , T_1 is moderately sensitive but s and T are insensitive c_1.
- s, T_1, T, Q, ρ, PE, π_1 *and* π are moderately sensitive to the parameter c_2,$N$is sensitive but *d(s)* is insensitive to the parameter c_2.
- N, s, T_1, T,*d(s)*, Q, ρ, PE *and* π are moderately sensitive to the parameter c_3 but $\pi_1$is sensitive to the parameter c_3.
- N, s, T_1,Q, ρ, PE, π_1 , π and *d(s)* are sensitiveto parameter c but T is moderately sensitive to the parameter c.
- N,s, T_1 , T ,Q, ρ, π and *d(s)* are moderately sensitive to the parameter α but PE and π_1 are sensitive to the parameterα .
- N, s, $T_1, T, Q, \rho, PE, \pi_1\ and\ \pi$ are sensitive to the parameter K_1 and *d(s)* is moderately sensitive to the parameter K_1.
- N, s, *d(s)*, $T_1, T, Q, \rho, PE, \pi_1\ and\ \pi$ are sensitive to the parameter α_1.

Managerial Implications

Through sensitivity analysis it is observed that the net present value profit is influenced by the major parameters like $r, H, c_1, c_2, c_3, c, \alpha, K_1\ and\ \alpha_1$where it is not effected by the parameter β.So it is necessary to look into these factors to obtain more profit in the manufacturing system with this present inventory model. From sensitivity analysis with respect to the major parameters the decision maker can control the market demand through the manipulation of selling price and he can frame a significant strategy for increasing profit.

VII. CONCLUSION

In this paper, an EPQ model is introduced which investigates the optimal replenishment quantity,unit selling price, replenishment cycle, promotional effort factor andthe total value net profit with finite planning horizon for deteriorating items. The model considers the impact of price dependent quadratic demand, , promotion, shortages and varying rate of deterioration. The model can be used for electronics and other luxury products which are more likely to have the above characteristics. This paper provides a useful property for finding the optimal net present value profit with finite planning horizon for deteriorating items with decline quadratic demand and compared numerically to the other EPQ model with decline quadratic demand and without allowing promotional activities. The economic replenishment quantity Q^* and net present value profit π^* for the present model were found to be more than that of the compared model respectively. Hence the utilization of selling price dependent declined quadratic demand and promotional effort factor make the scope of application broader. Lingo 13.0 version software is used to derive the optimal number of replenishment and unit selling price and promotional effort factor to maximize the total present value net profit.Further, a numerical example is presented to illustrate the theoretical results, and some observations are obtained from sensitivity analysis with respect to the major parameters controlling the market demand through the manipulation of selling price is an important strategy for increasing profit. This can be achieved by using the joint optimal replenishment, pricing strategy and promotional strategy developed in this study.

In the future study, it is hoped further incorporate the proposed model into several situations such as stochastic market demand, fuzzy decision parameters, partial back logging, and inflation.

VIII. REFERENCES

Abad, P.L., Optimal Pricing and lot sizing under conditions of perish ability and partial back ordering . *Management Science*, (1996), 42:1093-1104.

Bahari Kashani, H., Replenishment Schedule for deteriorating items with time proportional demand. *Journal of Operational Research Society*, (1989), 40:75-81.

Berrettoni, J.M., Practical applications of Weibull distribution. *Industrial Quality Control*, (1964), 21:71-76.

Bose, S., Goswami, S.B., Choudhuri, K.S., An EOQ model for deteriorating items with linear time dependant demand rate and shortages under inflation and time discount. *Journal of Operation Research Society*, (1993), 46:771-782.

Buzacott, J.A., Economic Order Quantities with inflation. *Operational Research Quarterly*, 26(3): 553-558.

Chandra, M.J., Bahner, M.L., The effects of inflation and time-value of money on same inventory systems. *International Journal of Production Research*, (1985), 23(4):723-730.

Chang, H.J., Dye, L.Y., An EOQ model for deteriorating items with time varying demand and partial backlogging. *Journal of Operational Research Society*, (1999), 50:1176-1182.

Chung, K.J., Ting, P.S., A heuristic for replenishment for deteriorating items with a linear trend in demand. *Journal of Operation Research Society*, (1993), 44:1235-1241.

Covert, R.P., Philip, G.C., An EOQ model for items with Weibull distribution deterioration. *AIIE Transactions*, (1973), 5:323-326.

Dave, U., Patel, L.K., (T, Si), Policy inventory model for deteriorating items with time proportional demand. *Journal of Operational Research Society*, (1981), 40:137-142.

Ghare, P.K., Schrader, G.F., A Model for exponential decaying inventory. *Journal of Industrial Engineering*, (1963), 14:238-243.

Goyal, S.K., Giri, B.C., Recent trends in modelling of deteriorating inventory. *European Journal of Operation Research*, (2001), 134:10-16.

Hariga, M.A., Effects of inflation and time value of money on an inventory model with time dependant demand rate and shortages. *European Journal of Operational Research*, (1995), 81:512-520.

Hollier, R.H., Mak, K.K., Inventory replenishment policies for deteriorating items in a declining market. *International Journal of Production Research*, (1983), 21:813-826.

Law, S.T., Inventory and Pricing policies for deteriorating items taking account of time-value. Unpublished Thesis. *Industrial Engineering Department, Chung Christian University*, (1997), .

Mishra, R.B., A note on operational Inventory Management under Inflation. *Naval Logistics Quarterly*, (1979), 26:161-165.

Pattnaik, M., A note on Optimal Inventory Policy involving Instant Deterioration of Perishable items with Price Discounts. *The Journal of Mathematics and Computer Science*, (2011), 3(4):390-395.

Pattnaik, M., Entropic Order quantity (EnOQ) Model under Cash Discounts. *Thailand Statistician Journal*, (2011), 9(2):129-141.

Pattnaik, M., Fuzzy NLP for single item EOQ model with Demand Dependent unit price and variable setup cost. *World Journal of Modeling and Smulations*, (2013), 9(1):74-80.

Pattnaik, M., Model of Inveontory Control. *Lambart Academic Publication, Germany*, (2012), .

Pattnaik, M., Note on Profit Maximization Fuzzy EOQ Models for Deteriorating items with two Dimension Sensitive Demand. *International Journal of Management Science and Engineering Management*, (2013), 8(4):229-240.

Pattnaik, M., Optimization in an Instantaneous Economic order quantity (EOQ) model incorporated with promotional effert cost, variable ordering cost and unit lost due to deterioration. *Uncertain Supply Chain Management*, (2013), 1(2):57-66.

Pattnaik, M., Wasting of Percentage on hand inventory of an Instantaneous EOQ model due to Deterioration. *Journal of Mathematics and Computer Science*, (2013), 7(3):154-159.

Sachan, R.S., On (TS) Policy inventory model for deteriorating items with time proportional demand. *Journal of Operational Research Society*, (1984), 35(11):1013-1119.

Salameh, M.K., Jaber, M.Y. and Noueihed, N. “Effect of deteriorating items on the instantaneous replenishment model”. *Production Planning and Control*, (1993), 10(2): 175-180.

Sarkar, B.R., Pan, H., Effects of inflation and time value of money on order quantity and allowable shortage. *International Journal of Production Economics*, (1994), 35:65-72.

Tripathy, P.K., Pattnaik, M. and Tripathy, P. “Optimal EOQ Model for Deteriorating Items with Promotional Effort Cost”. *American Journal of Operations Research*, (2012), 2(2): 260-265.

Tsao, Y.C. and Sheen, G.J. “Dynamic pricing, promotion and replenishment policies for a deteriorating item under permissible delay in payment”. *Computers and Operations Research*, (2008), 35: 3562-3580.

Wee, H.M., A deterministic lot size inventory model for deteriorating items with shortages and a declining market. *Computers Operations Research*, (1995), 22(3):345-356.

Wee, H.M., A replenishment policy for items with a price dependent demand a varying rate of deterioration. *Production Planning and Control*, (1997), 8(5):494-499.

Wee, H.M., Law, S., Replenishment and Pricing policy for deteriorating items taking into account the time-value of money. *International Journal of Production Economics*, (2001), 71:213-220.

Wee, W.H., Economic Production lot size model for deteriorating items with partial back ordering. *Computer and Industrial Engineering*, (1993), 24(37):449-458.