Tradeoff Ordering Policy and Decision biased Newsvendor with Uncertain Demand

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This paper addresses two problems: (1) how to determine order quantity when buyer's risk bias affects her decision and no reliable demand estimate is available because of high fashion content; (2) how much to order when demand variance is very high and only one-shot inventory decision is possible. We propose an ordering policy when demand is fuzzy and buyer's decision trades off between waste-averse and stockout-averse preferences. We demonstrate that buyer is more conservative for high margin products and more aggressive for low products than classic newsboy with uncertain demand. Our second policy involves a contract with single manufacturer and a buyer. Manufacturer offers a discounted price for a time span when demand variance is high. At the time of ordering, buyer needs to tradeoff between shortage- penalty-cost and reliable demand estimate as, her demand information is only a conjecture about mean and variance. We extend the previous research on worst case distribution and derive an optimal ordering policy. Numerical results show that buyer's risk profile makes her to order aggressively when demand uncertainty is high.
Keywords Decision bias; uncertain demand; Newsboy problem; Time varying variance,; Worst case distribution

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I. INTRODUCTION

The newsvendor applies to array of settings, for example for fashion apparels when buyer must order before selling season and without any opportunity for replenishment during the season. Short selling seasons are common for trendy and high fashion apparel industries as the catalog customers always face the dilemma of balancing the stockout and the inventory holdup. This single period inventory policy is typically a newsboy problem in an uncertain environment. There are many a businesses such as fashion, sporting and services that always face uncertain demand and short selling single season. Buyer is always facing the dilemma of how much quantity to order because she does not want either the back orders or extra inventory stock problem.

Newsvendor decisions can have significant consequences. Ziegler(1994, 1995) mentioned that IBM produced more than $700 million of excess inventory one year and in another year lost potential revenue of $100 million for under producing their Aptiva PC line. Similarly, for some restaurants and fast food chains, underestimating demand is very common. It shows that managers face difficulty in making such decisions and mere newsvendor may not suffice for every situation. Thus an ordering policy, for a newsboy type problem is very important for designing any contract that takes into account the combined bias and uncertainty simultaneously.

Hadley and Whitin (1963) are the first to develop a solution to the newsboy problem by applying numerical methods and solving with dynamic programming. Gallego and Moon (1993) analyze the worst case distribution as an effective ordering policy when buyer can only make an educated guess about the mean and variance of the item.

This paper seeks to aid buyers in one-shot inventory decisions before the start of selling season. These decisions may systematically deviate from profit maximization for several reasons as buyer may have risk preferences. For example, she may order less because of extra inventory averse behavior or may order more if she is stockout-averse. We assume that buyer has no historical data available unlike Fisher and Raman (1996) who suggest that although styles change in apparels but closely resembles demand distribution of similar styles throughout many years. Such resemblance carries high demand variability and fast fashion apparels even do not exhibit similar trends because of high product differentiation and fashion contents, (Doeringer and Crean, 2005). We assume a buyer who can only estimate mean for otherwise a fuzzy demand for her product. Firstly, we propose an ordering policy based on combined bias and fuzzy demand distribution. We show that such policy is more efficient as it only requires estimated mean and combined bias makes it more effective. Such policy depicts a case of trendy, high fashion products with a very short season and buyer is more concerned about her risk profile because of limited distributional information. Biased behavior can be stockout averse or waste-averse.

Here, waste-averse means reluctance to pile extra inventory. We model it and show that it depends upon parameters for understocking and overstocking cost. Moreover we define uncertainty demand distribution by fuzzy numbers.

Secondly, we entail the ordering policy for the buyer in a contractual setting. Buyer's knowledge about demand is only limited to an educated guess about mean and variance. The demand season is short and only one-shot inventory decision is possible because variance is higher at the start of the selling season. Manufacturer proposes a contract at the start of time span (0, T) and encourages early ordering let's say at time t before the stipulated ordering time by offering a discounted wholesale price. If buyer places order at or after T, she pays a regular price. We determine the optimal order quantity taking into account the impact of time-varying variance and ordering time. We use results from Gallego and Moon (1993) to derive the optimal quantity based on worst case distribution. Our results differ from the classical approach as we use time sensitive variance and ordering along with discounted price mechanism. We also assume shortage cost which could be effective in determining the risk preferences of the buyer. Following assumptions are valid for such contract and ordering policy:

1. All the firms possess the same information about the parameters like wholesale price and salvage value of the product.

2. Demand is fuzzy with deterministic wholesale and retail price.

3. Manufacturer is risk neutral.

4. Buyer has both stockout averse and waste averse preferences.

1.1. Notations used:

P = Retail price
c = Production cost
w = Wholesale price
υ = Salvage value for buyer
x = Random variable
α = Cost of overstock
β = Cost of understock
ϕ = Fuzzy demand distribution
$\wedge$ = Min operator
T = Max time for purchase at w'
w' = Discounted wholesale price
λ = Known constant that adds to w' after time T

μ = Mean demand

σ = Standard deviation of demand

We present the related literature in section 2. Section 3 comprises of the proposed ordering policy with demand fuzziness and buyer's risk profile, and also exhibits an ordering policy when buyer minimizes her stocking risk by assigning shortage-penalty-cost and time-varying demand variance. We include this ordering behavior into a contractual relationship when manufacturer entices buyer to order more well before demand forecast by offering a discounted wholesale price. Section 4 summarizes the results and numerical study with managerial insight.

II. RELATED LITERATURE

Single period newsvendor problem is a significant scenario for products with limited selling season. Fashion apparels, sporting and service industries require booking in advance. Gallego and Moon (1993) define the newsboy for a single period one time inventory decision by worst case distribution. They assume random demand and a buyer with only an educated guess of demand mean and variance. Lau and Lau (1996) model the multiproduct capacitated newsboy for achieving a target profit level. Khouja (1999) suggests a comprehensive review of the single period inventory problem and its extensions in detail.

Most of the research however has been done with stochastic demand based on probability theory. Petrovic and Petrovic (1996) present two models for demand and imprecise costs modeled using fuzzy set theory. Li and Kabadi (2002) also present two models in which they use probabilistic demand with fuzzy costs in one and in other assume fuzzy demand and deterministic costs. They use fuzzy ordering numbers to find optimal order quantity. Our proposed ordering policy differs as we use buyer's risk preferences along with uncertain demand distribution to obtain optimal ordering quantity. Our model is in line with findings of Schweitzer and Cachon (2000) that use experimental findings and suggested that decision maker systematically deviates from profit maximizing quantity. They show that decision bias affects by reducing difference between ordered quantity and realized demand.

There has been a lot of research on distribution-free newsboy problem when buyer only assumes an educated guess about the mean and variance. Scarf (1958) pioneers the minimax approach which minimizes the maximum cost resulting from the worst possible distribution. Gallego and Moon (1993) provide extended results for the Scarf's ordering rule to random yields, fixed ordering cost and constrained multiple products along with recourse case when second ordering opportunity. Moon and Silver (2000), develop distribution-free heuristics for a multi item with a budget constraint and fixed ordering costs.

There are also numbers of papers which deal with uncertain demand and forecast updates. Murray and Silver (2006) use Bayesian approach for estimating demand. Love joy (1990); develop a myopic strategy using a parameterized adaptive demand process. Sethi and Sorger (1991) develop a dynamic programming framework for rolling horizon decision making with forecast update but bearing some additional cost. They develop a stochastic production problem requiring forecast window and optimal production quantity in each period.

Our model works with a single shot inventory ordering when forecast update is not possible as demand process is fuzzy in nature. We extend the work of Gallego and Moon (1993), in the second part of this paper. We provide the optimal order quantity by minimizing the cost against the worst possible distribution. Our policy entails a contract which allows a time sensitive variance and purchasing decision along with incorporation of shortage cost. However, the main focus of this work is on the ordering policies for the buyer who is faced with an uncertain demand and reliable demand forecast is available only at some future time.

III. MODEL FORMULATION

The proposed strategy takes into account the risk elements when buyer has an extra burden of salvaging extra units or minimizing the potential lost sales. The chaos created by uncertain demand is modeled by fuzzy set theory and buyer's waste averse and stockout averse behavior are incorporated in newsvendor to obtain the optimal quantity.

Our second approach encompasses an ordering policy when distributional information is limited for the buyer at the time of purchase. Manufacturer offers a contract which allows the buyer to purchase at a discounted price if she orders in a given time span (0, T). Demand information is only available after time T but then buyer purchases at a price of $w+\lambda$. We assume that only one shot inventory decision is possible and buyer takes advantage of the discounted price. Her demand information is no more than an educated guess about demand mean and variance. Buyer's risk profile contains a penalty cost for shortage and high demand variance because of uncertainty at the decision epoch. We determine the optimal order quantity against the worst possible scenario with the objective of minimizing overall cost rather than maximizing the total profit for the buyer.

"Fig. 1" shows the decision epochs for the buyer who trades off between her risk profile and profit making opportunity given by the manufacturer. Main objective for this contract is to determine one shot quantity during the time span (0, T).

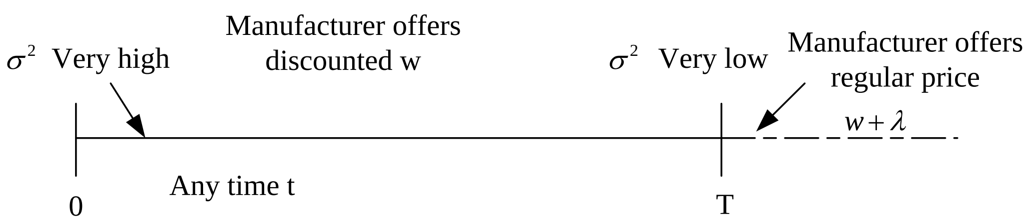


Fig 1. Decision epoch for buyer under the contract

3.1. Preliminaries

Remark 1: An uncertain variable ξ is said to have a first identification function λ if (i) $\lambda(x)$ is a nonnegative function on $\Re$ such that $\sup_{x \neq q}(\lambda(x)+\lambda(q)) = 1$; (Liu, (2007))

(ii) For any set of **B** real numbers, we have

$$M\{\xi \in B\} = \begin{cases} \sup \lambda(x) & if \sup_{x \le q} \lambda(x) < 0.5 \\ 1-\sup \lambda(x) & if \sup_{x > q} \lambda(x) \ge 0.5 \end{cases}$$

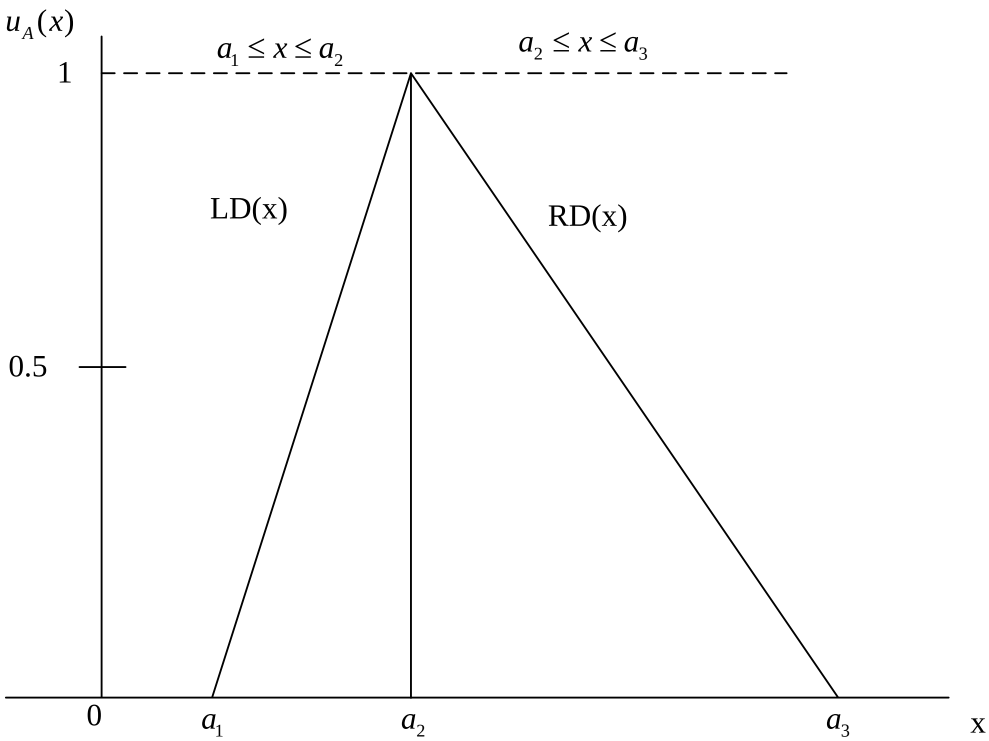
Remark 2: According to Liu, (2009); let ξ be an uncertain variable. Then the expected value of ξ is defined by:

$$E|\xi| = \int_0^{\infty} M\{\xi \ge x\}dx - \int_{-\infty}^{0} M\{\xi \le x\}dx$$

Provided that at least one of the two integrals is finite

Remark 3: For a single period inventory problem where demand is subjective and represented by a generalized triangular fuzzy number with a following membership function

$$u_A(x) = \begin{cases} 0 & x \le a_1 \\ \dfrac{LD(x)}{2} & a_1 \le x \le a_2 \\ \dfrac{RD(x)}{2} & a_2 \le x \le a_3 \\ 1 & x \ge a_3 \end{cases}$$



Triangular Fuzzy Number

Fig 2. Membership function of a Triangular fuzzy number and associated ordinary numbers.

3.2. Optimal Ordering Policy with Combined Risk Bias

We propose a combined bias with uncertain fuzzy newsboy setting for optimal ordering. This setting does not require estimate of demand variance and only cost parameters suffice. This scheme suggests a systematic ordering policy when single period demand is highly uncertain involving buyer's risk aversion for unsold inventory and stockout averse factors as well. Such policy allows buyer to maximize her profit and utility simultaneously.

Proposition 1 *When* $\dfrac{P-w+\beta}{P-\upsilon+\alpha+\beta} < \dfrac{P-w}{P-\upsilon}$, *buyer's extra inventory averse preference is more than stockout-averse preference and vice versa.*

Following hold true:

1. For product i.e.; with high critical fractile, quantity demanded by combined bias is always lesser than quantity with classic newsboy with uncertain demand.
2. For low product i.e.; with low critical fractile, quantity demanded by combined bias is always

more than quantity with classic newsboy with uncertain demand.

3. For high product, buyer orders more conservatively with combined bias than with classic newsboy and vice versa.

Lemma 1:

A unique optimal order quantity exists for the buyer that maximizes her utility with an uncertain demand. Such q^* is:

$$q^* = \phi^{-1}\left(\frac{P-w+\beta}{P-\upsilon+\alpha+\beta}\right) \qquad (1)$$

Proof: See Appendix

We can easily conclude that buyer's quantity increases with increasing stockout preference and decreases with increasing waste-averse preference. Such result is valid for any probability distribution as well, because of its dependence on parameters. However, in the absence of data or where similar item differs in contents to such an extent that demand variance may not be the best indicator of demand variability; fuzzy demand gives better cushion to buyer. For uncertainty distribution buyer requires minimum, most likely and maximum subjective estimates only.

Lemma 2:

For a triangular uncertain variable ξ, the identification function corresponds to

$$\lambda(x) = \begin{cases} \dfrac{x-a_1}{2(a_2-a_1)} & a_1 \le x \le a_2 \\ \dfrac{x-a_3}{2(a_2-a_3)} & a_2 \le x \le a_3 \end{cases}$$

Where a_1, a_2 and a_3 are real numbers and $a_1 < a_2 < a_3$, optimal order quantity

(1) When $\dfrac{P-w+\beta}{P-\upsilon+\alpha+\beta} < 0.5$ is

$$q^* = \frac{(2w-\beta-P-\upsilon+\alpha)a_1 + 2(P-w+\beta)a_2}{P-\upsilon+\alpha+\beta}$$

(2) When $\dfrac{P-w+\beta}{P-\upsilon+\alpha+\beta} \ge 0.5$ is

$$q^* = \frac{2(w-\upsilon+\alpha)a_2 - (2w-\upsilon+\alpha-P-\beta)a_3}{P-\upsilon+\alpha+\beta}$$

Proof: See appendix.

Above results show that buyer's risk increases with higher inventory and she orders conservatively for high products such as fashion apparels and trend setters. However, for low margin profit products she orders more aggressively even if compared to a newsvendor without bias. The proposed policy is effective as we can get a closed form solution and gives better insight about the effect of model parameters.

3.2.1 Numerical Illustration

Let c= 0.5, w= 4, P= 10, υ = 3, α = 7 and β = 6 with a_1=2200, a_2= 2500 and a_3= 2600. It is easily verifiable that for low critical fractile, classic newsboy with uncertain demand is $q^* = \dfrac{(2w-P-\upsilon)a_1 + 2(P-w)a_2}{P-\upsilon}$. Similarly for high critical fractile it is given by $q^* = \dfrac{2(w-\upsilon)a_2 - (2w-\upsilon-P)a_3}{P-\upsilon}$. We compare both policies numerically and critical fractile and order quantity for classic newsboy are 0.86 and 2571 units respectively compared to 0.60 and

2520 units for the proposed policy. For high products such implication holds true for feasible range of parametric values. Likewise for low products, order quantity with classic newsboy is lesser than the order quantity with the proposed policy.

3.3 Tradeoff Ordering Policy

Our second ordering policy entails a contract that allows buyer to make purchase at a discounted wholesale price. At the start of the season buyer's risk is at maximum and she delays to the point where she gets better demand forecast to resolve demand variance. For a high fashion and trendier products such delay reduces the overall lead time for the manufacturer. He offers a contract with a discounted wholesale price for the time span (0, T). For this time span, buyer can only make use of an educated guess about demand mean and variance. Especially demand variance is very high during that ordering period. Here we discuss the typical newsvendor type scenario when buyer determines her order quantity for such discounted price and also taking into account the shortage cost. We extend Gallego and Moon (1993) results under time sensitive variance and shortage cost with buyer minimizing her maximum cost against worst possible distribution.

3.3.1 Notations used

Let us define notations for this section as:

P = (1+m) $w^{'} > w^{'}$ Retail price

υ = (1-d) $w^{'} < w^{'}$ Salvage value for buyer

k = Penalty cost for shortage

$l = \frac{k}{w^{'}}$ Penalty cost fraction

Following holds true for the proposed ordering policy.

Proposition 2 *There exists a unique order quantity Q^* that minimizes buyer's cost against the worst possible distribution.*

The optimal order quantity is:

$$Q^* = \mu + \frac{(1-\frac{t}{T})\sigma}{2}\left[\left(\frac{l+m}{d}\right)^{\frac{1}{2}} - \left(\frac{d}{l+m}\right)^{\frac{1}{2}}\right] \qquad (2)$$

To verify whether Q^* is strictly convex, we can compute the second order derivative w.r.t Q:

$$\frac{(d+m+l)(1-\frac{t}{T})^2\sigma^2}{2\left((1-\frac{t}{T})^2\sigma^2 + (Q-\mu)^2\right)^{\frac{3}{2}}} > 0$$

Q^* is the optimal quantity that accounts for shortage cost as well as time sensitive demand variance.

3.3.2 Model Derivation

The buyer maximizes her profit function as given by

$$\Pi_R = P(Q \wedge D) + \upsilon E(Q-D)^+ - kE(D-Q)^+ - (w-\lambda)Q$$

Follow results from Gallego and Moon (1993) we get

$$(Q \wedge D) = D - (D-Q)^+$$

$$(Q-D)^+ = (Q-D) + (D-Q)^+$$

$$(D-Q)^+ = (D-Q) + (Q-D)^+$$

Thus the profit function can be written as

$$\Pi_R = (P-\upsilon)\mu - (P-\upsilon)E(D-Q)^+ - (w^{'}-\upsilon)Q - kE(D-Q)^+$$

Or put it differently

$$\Pi_R = (P-\upsilon-k)\mu - (P-\upsilon)E(D-Q)^+ - (w^{'}-\upsilon-k)Q + kE(Q-D)^+$$

We can minimize the objective function in terms of cost:

$$\text{Min} \quad w^{'}\left[(m+d-l)\mu - (m+d)E(D-Q)^+ - (d-l)Q - lE(Q-D)^+\right]$$

We used the ***lemma 1*** from Gallego and Moon (1993), and extended

$$E(D-Q)^+ \le \frac{\left((1-\frac{t}{T})^2\sigma^2 + (Q-\mu)^2\right)^{\frac{1}{2}} - (Q-\mu)}{2}$$

Similarly

$$E(Q-D)^{+} \leq \frac{\left((1-\frac{t}{T})^{2}\sigma^{2}+(Q-\mu)^{2}\right)^{\frac{1}{2}}+(Q-\mu)}{2}$$

We can rewrite the cost function as

$$\text{Min} \quad (d-l)Q+(m+d)\left[\frac{\left((1-\frac{t}{T})^{2}\sigma^{2}+(Q-\mu)^{2}\right)^{\frac{1}{2}}-(Q-\mu)}{2}\right]+l\left[\frac{\left((1-\frac{t}{T})^{2}\sigma^{2}+(Q-\mu)^{2}\right)^{\frac{1}{2}}+(Q-\mu)}{2}\right]$$

And by FOC of optimality we can derive the optimal order quantity.
We can conclude with following insight for this ordering policy.
Proposition 3 *With time sensitive variance and shortage cost combined, buyer's order quantity increases with penalty cost and is decreasing when time elapsed for ordering increases*.
We can demonstrate proposition 3 by numerical study.

3.3.3 Numerical Insight

We compare the proposed strategy with the benchmark case from Gallego and Moon (1993).The optimal order quantity with fixed demand variance and without penalty cost is given by:

$$Q^{*}=\mu+\frac{\sigma}{2}\left[\left(\frac{m}{d}\right)^{\frac{1}{2}}-\left(\frac{d}{m}\right)^{\frac{1}{2}}\right] \qquad (3)$$

This is a well known result from Gallego and Moon (1993). For numerical analysis we assume following data

$$\mu=1000$$
$$\sigma=200$$
$$w^{'}=10$$
$$P=30$$
$$k=15$$
$$\upsilon=5$$
$$m=\frac{P}{w^{'}}-1=2$$
$$d=1-\frac{\upsilon}{w^{'}}=0.5$$
$$l=\frac{k}{w^{'}}=1.5$$
$$t=5$$
$$T=10$$

The benchmark quantity from (3) is 1150 units whereas from (2) it is 1226 units when t=0 and 1113 units when t=5.
Note that time varying impact of demand variance and penalty cost affect order quantity differently. We verify the impact separately for in-depth understanding of the ordering behavior. "Fig. 3" shows the behavior of optimal order quantity with increasing ordering time.

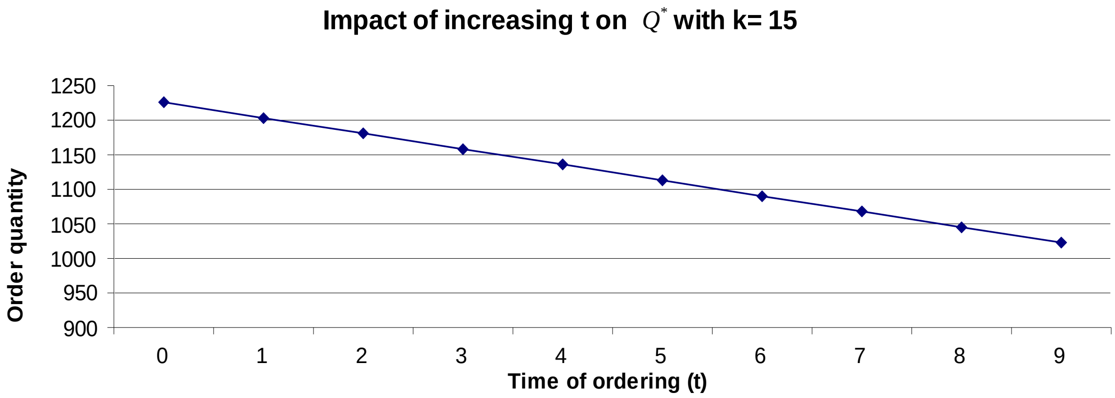


Fig 3. Impact of varying ton optimal order quantity for the buyer

We can infer that the buyer needs more quantity protection when she orders early as reliable demand forecast is not available and if she orders later, she orders considerably lesser as more reliable demand estimate becomes available.

On the other hand if unit penalty cost increases, buyer orders aggressively as she requires more protection against shortage. Thus time varying variance, penalty cost and ordering time are important factors and have significant impact on ordering decision under the defined scenario

IV. CONCLUDING REMARKS

In this paper we first propose an ordering policy with buyer's risk defined by a penalty cost for shortage and salvage cost for overstock. We show that, along with these parameters, demand uncertainty is also a main factor which affects ordering policy. Buyer orders aggressively or conservatively according to the risk assigned by these factors. Demand for high fashion and trendy products is not easy to estimate even from similar items because of different utility these items incur on the customers. We assume a fuzzy demand scenario with buyer's risk profile and show that buyer‘s behavior is different than classic newsboy problem. For high critical

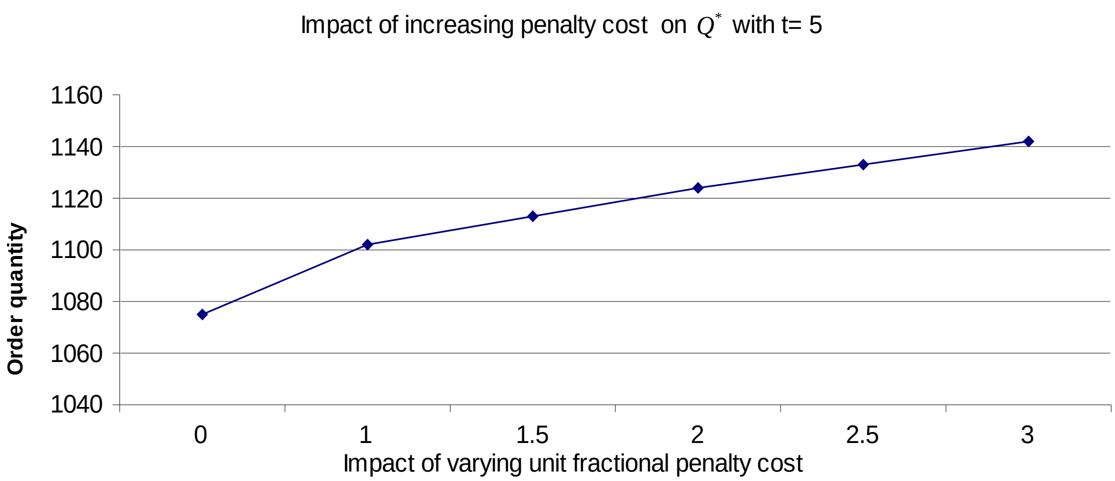


Fig 4. Impact of varying unit fractional penalty cost (L) on optimal order quantity

fractile, buyer is more conserve in ordering than a newsboy with uncertain demand and vice versa is essentially true for low critical fractile. Such ordering policy is very helpful in the absence of data and only requires minimum, most likely and maximum estimate of demand. So demand variance and mean are not at all required for such a subjective treatment.

We next identify some important features of a contract that allows buyer to purchase at a discounted price but time span is limited and demand is the most uncertain at that time. Demand variance is very high and buyer is stockout averse. Here we compute an efficient ordering policy that extends previous work by Gallego and Moon (1993). We employ time sensitive variance and penalty cost for determining optimal quantity under worst possible scenario. Our policy actually, is a tradeoff which a buyer makes between profit and cost. We entail the policy in a contract whose features can be discussed in detail and extended for multiple seasons. Finding important factors and efficiency of the contract is a new avenue of research.

Another future research would be defining an ordering policy for multiple periods and for multiple items. The highlight for such research could be the ordering behavior of buyer when she sees different epoch to order for a single period or multiple periods.

Acknowledgments

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Appendix

Proof for Lemma 1:

For continuous uncertain distribution $\phi(x)$, self dual exists for uncertain measure.

Let the buyer maximizes her expected profit/utility function:

$$(P-\upsilon)S(q)-(w-\upsilon)q-\alpha\int_0^q (q-x)M\{\xi \le x\}dx-\beta\int_q^x (x-q)M\{\xi.>x\}dx$$

$$(P-\upsilon)(q-\int_0^q (q-x)M\{\xi \le x\}dx)-(w-\upsilon)q-\alpha\int_0^q (q-x)M\{\xi \le x\}dx-\beta\int_q^x (x-q)(1-M\{\xi.>x\}dx)$$

$$\frac{\partial \Pi_R}{\partial q}=(P-\upsilon)-(P-\upsilon)M\{\xi \le x\}-(w-\upsilon)-\alpha M\{\xi \le x\}+\beta-\beta M\{\xi \le x\}$$

$$M\{\xi \le q^*\}=\frac{P-w+\beta}{P-\upsilon+\alpha+\beta}$$

$$q^*=\phi^{-1}\left(\frac{P-w+\beta}{P-\upsilon+\alpha+\beta}\right)$$

This completes the proof.

Proof for Lemma 2:

The corresponding membership function for a triangular fuzzy number is defined by a triplet A = (a_1, a_2, a_3)

We can show that

$$u_A(x) = \begin{cases} \dfrac{x-a_1}{(a_2-a_1)} & a_1 \le x \le a_2 \\ \dfrac{x-a_3}{(a_2-a_3)} & a_2 \le x \le a_3 \end{cases}$$

And for case 1 and case 2 according to **Remark 1**, **Remark 2** and **Remark 3** we can analyze the credibility value of fuzzy demand for the two cases

$$M\{\xi \le q\} = \frac{LD(x)}{2} = \frac{P-w+\beta}{P-\upsilon+\alpha+\beta}$$
$$M\{\xi \le q\} = \frac{RD(x)}{2} = 1-\frac{P-w+\beta}{P-\upsilon+\alpha+\beta}$$

This corresponds to
$$\sup_{x\le q^*} \lambda(x) = \sup_{x\le q^*} \frac{x-a_1}{2(a_2-a_1)} = \frac{P-w+\beta}{P-\upsilon+\alpha+\beta}$$
$$\sup_{x> q^*} \lambda(x) = \sup_{x> q^*} \frac{x-a_3}{2(a_2-a_3)} = 1-\frac{P-w+\beta}{P-\upsilon+\alpha+\beta}$$

The corresponding identification function is:

$$\lambda(x) = \begin{cases} \dfrac{x-a_1}{2(a_2-a_1)} & a_1 \le x \le a_2 \\ \dfrac{x-a_3}{2(a_2-a_3)} & a_2 \le x \le a_3 \end{cases}$$

And following immediately follows for case 1

$$M\{\xi \in q^*\} = \frac{P-w+\beta}{P-\upsilon+\alpha+\beta}$$
$$\frac{q^*-a_1}{2(a_2-a_1)} = \frac{P-w+\beta}{P-\upsilon+\alpha+\beta}$$

The optimal order q is
$$q^*-a_1 = \frac{2(w-\beta-P)a_1+2(P-w+\beta)a_2}{P-\upsilon+\alpha+\beta}$$
$$q^* = \frac{(2w-\beta-P-\upsilon+\alpha)a_1+2(P-w+\beta)a_2}{P-\upsilon+\alpha+\beta}$$

For case 2

$$\frac{q^* - a_3}{2(a_2 - a_3)} = 1 - \frac{P - w + \beta}{P - \upsilon + \alpha + \beta}$$
$$\frac{q^* - a_3}{2(a_2 - a_3)} = \frac{w - \upsilon + \alpha}{P - \upsilon + \alpha + \beta}$$

The optimal order quantity is given by

$$q^* = \frac{2(w - \upsilon + \alpha)a_2 - (2w - \upsilon + \alpha - P - \beta)a_3}{P - \upsilon + \alpha + \beta}$$

This completes the proof.