Daily Aircraft Maintenance Routing Using a Deficit Function Hollow Graph

Helman I. Stern
Ben Gurion University of the Negev, Beersheba, Israel

Robert M. Saltzman*
San Francisco State University, San Francisco, California, USA

We solve the aircraft maintenance routing problem (AMRP), with rotations for a homogeneous fleet and a repeated daily flight schedule (FS). We develop a new, more compact flight connection graph called a hollow graph. Flight connection opportunities are derived from hollow regions between maximum values of a deficit function representation of the FS. They allow a determination of the minimum aircraft fleet without need for a plane count constraint. We solve the AMRP as a dual-objective multicommodity integer linear program. The paper's main contributions are: (1) integration of the minimum fleet size in the AMRP, (2) use of a deficit function representation of the FS, (3) reduction of aircraft connection graph and number of commodities, resulting in reduced problem size and faster run times, and (4) a dual-objective AMRP. Compared to existing formulations with larger fleets and connection networks we obtain much smaller problem sizes that execute up to 95% faster.

* Corresponding Author. E-mail address: saltzman@sfsu.edu

I. INTRODUCTION

One of the most studied problems in aircraft scheduling is the aircraft maintenance feasible routing problem (AMRP). Given a flight schedule (FS) for a fleet of identical aircraft types, the AMRP consists of building aircraft routes that respect maintenance requirements and cover each flight in the FS exactly once. We assume all flights in the FS start and end within a finite period of length T, namely, a daily schedule repeated weekly. A set-partitioning or set-covering based formulation to solve the AMRP is the most commonly used approach. A route generator is used to obtain a large number of candidate routes. These are introduced as columns of an integer set partitioning problem such that each flight is covered exactly once. Another popular method is an integer multi-commodity network flow (MCF) problem formulation, mostly based on an underlying flight connection network. While we also use a MCF formulation, the difference in our approach is that we provide a predetermination of the minimum fleet size required to service the aircraft schedule of flights and integrate it with the standard formulation. Other authors allow the fleet size to be bounded by the aircraft carrier's existing fleet size. We eliminate this "plane count constraint" which allows solutions with more than the necessary fleet size, incurring unnecessary increased capital costs. Thus, our approach minimizes one of the largest costs of a commercial airline, the aircraft themselves. Interestingly, there are very few published papers on how airlines minimize this capital cost. Moreover, we institute a dual-objective function, whereby we can maximize either the number of "balanced" routes in which aircraft return to the terminals from whence they started or the number of "maintenance feasible" routes in which aircraft undergo a maintenance operation performed overnight.

An aircraft route is a sequence of connecting flights. In order for two flights to be connected two main conditions must be satisfied: 1) the flights must arrive and depart at the same airport and 2) the first flight must arrive before the scheduled departure time of the second flight. Aircraft connection opportunities are derived from a deficit function (DF) representation of a flight schedule (Linis and Maksim, 1967), (Gertsbakh and Gurevich, 1977). Each DF is associated with an aircraft terminal and is a step function that increases and decreases by one at flight departure and arrival times, respectively. The DF admits to the classical minimum fleet size theorem that determines the minimum number of aircraft needed to service the FS. In addition, it finds the number of aircraft routes that start and end at each terminal. DFs exhibit regions of maximal values between which are valleys referred to as *hollows*. Analysis of these hollows reveals that they are composed of the same number of flight arrivals and departures, which offer many flight connection possibilities. This observation allows us to develop a new type of connection graph referred to as a *hollow graph*. Hollow graphs are more compact than connection graphs used by others as only a limited number of specific connections are allowed. This forms the basis of a new integer linear multicommodity network formulation. The main contributions of this paper are the construction of a new type of connection network based on the minimum fleet size and a reduced set of connection arcs. This reduction is a result of an extended flight connection rule. In addition, unlike other works, our formulation of the AMRP finds and maintains the minimum fleet without the need to add a plane count constraint. Our formulation also uses fewer commodities than those of other MCF airline routing formulations. Consequently, we provide a smaller and more compact mathematical formulation of the AMRP. We test our formulation against other forms of connection network reduction methods to demonstrate the advantage of ours. Although this investigation addresses the airline schedule problem, it has applications to other domains as well, such as ground transportation and even job-machine scheduling. In summary, our main contributions are (1) a new methodology – the first use of the DF approach for aircraft maintenance routing; (2) integration of the minimum fleet size into the AMRP; (3) graph reduction, *i.e.*, using a compact connection graph that is more efficient than those used by previous researchers; and (4) a new objective formulation, *i.e.,* a dual-objective MCFP that can be used to maximize either the number of maintenance feasible routes or the number of balanced routes.

The following section provides a short review of the aircraft routing literature including the deficit function approach. Section 3 introduces the deficit function representation of a flight schedule and its role in the classical minimum fleet size theorem. It also describes the importance of deficit function *hollow regions* and their role in feasible flight connections. Section 4 formulates the ARMP using a special compact flight connection hollow graph, while in section 5 the method is demonstrated on an example problem. A performance evaluation is presented in section 6. The paper closes in section 7 with a conclusion and future work.

II. REVIEW OF AIRCRAFT ROUTING

Finding good solutions to the AMRP is

challenging, and many approaches have been taken. Lacasse-Guay, Desaulniers, and Soumis (2010) and Maher, Desaulniers, and Soumis (2014) both review three main variations for solving the ARMP, *e.g.,* strings, big-cycle, and one-day routes. A string is a sequence of connected flights that starts just after a maintenance check and ends with the next maintenance check. It can cover several days of flight. A paper by Xu, et al. (2019) considers a stochastic tail assignment under recovery problem, which is based on the integer flight string model of Barnhart, et al. (1998). In the big-cycle approach, routes are determined for a daily cyclic schedule. Each aircraft is assigned to this cycle with a single period delay between each aircraft's route. The goal of the approach is to equalize aircraft utilization. This method is most applicable to FSs that have repeated daily schedules. The cycle is constructed to be maintenance feasible by scheduling maintenance visits at required daily intervals. The routes obtained from using the string and big cycle variants do not consider the high probability that disruptions will occur during the operations. This factor is included in the one-day routes approach, which attempts to alleviate the effects of schedule disruptions from preceding days. It tries to maximize the number of routes that end up at a maintenance station at the end of one day. This stochastic treatment of the problem is a purely operational approach to the routing problem as opposed to the more tactical approaches of the string and big cycle that concatenate one-day routes to form complete routes.

The minimum fleet size problem was solved by the DF approach. DFs were introduced by Linis and Maksim (1967), Gertsbakh and Gurevich (1977), and Gertsbakh and Gurevich (1982) for airline scheduling. In addition, DFs have been used for scheduling of ground transport vehicles, *i.e.,* buses and trains (Liu and Ceder, 2017), for machine job scheduling (Gertsbakh and Stern, 1978), and for aircraft and crew scheduling (Gertsbakh and Stern, 2017), Stern and Gertsbakh, 2019). The DF approach has lost favor since the 1980s to solve the more complicated airline routing problem. We resurrect the DF and use it in a new formulation to solve the AMRP problem.

Haouari et al. (2013) describe an aircraft routing graph with flight legs as nodes and feasible connections as arcs. The routing problems imbedded in their integrated models include a plane count constraint, whereby the number of aircraft in the fleet is given *a priori*. This is in contrast to our DF approach, where we make no *a priori* assumption on the number of aircraft available, but instead find the minimum number of aircraft needed to service the FS and integrate this into the AMRP. Khaled et al. (2018) formulate the problem as a binary linear programming with a connection network. Unlike our approach they do not require the aircraft routes to be cyclic. We found Cordeau et al. (2001) to be the closest work to ours. Starting with a plane count constraint for a very large fixed fleet size they introduce a procedure to reduce it to obtain the minimum possible fleet. Using this fleet size as a constraint, they find a set of feasible origin-destination paths and construct a path-node coefficient matrix to be solved for the minimum number of paths in a set covering problem. Theirs is an interesting approach, but much more cumbersome than ours for finding the minimize fleet size.

III. DEFICIT FUNCTION, HOLLOWS, AND FEASIBLE FLIGHT JOININGS

3.1. Deficit Functions and Minimum Fleet Size

Let I={i: i=l, . . ., n} denote the set of required flight legs. Flights are flown between a set of terminals (airports) K={k: k=l, . . ., q}. A homogeneous fleet is assumed allowing any aircraft to carry out any flight, and each flight must be serviced by a single aircraft. For a flight i departing from terminal k^i_d and arriving at terminal k^i_a, let t^i_d and t^i_a represent its departure and arrival times, respectively. A flight leg i is represented as a quadruple (t^i_d , t^i_a, k^i_d, k^i_a), where $k^i_d \neq k^i_a$. A flight schedule FS is a set of all flight legs $\{i \in I\}$. Here, the arrival time has been prolonged to include the minimum turn time between any two flights. Denote [0, T] as a daily schedule horizon where flights are excluded from crossing the 24:00 hour line. All flights depart and arrive within this time interval. Such a daily FS is said to be *balanced*.

A set of DFs may be constructed for a balanced FS. A deficit function is a step function DF(k, t) associated with a particular terminal k, and defined over a schedule horizon [0, T]. DF(k, t) increases by 1 if a flight departs at time t, and decreases by 1 if a flight arrives at time t. For a balanced FS, DF(k, 0) = DF(k, T) = 0. Thus, DF(k, t) = number of flights departing – number of flights arriving in [0, t]. DFs contain multiple regions or intervals of maximal values (plateaus). Regions between plateaus are referred to as *hollows*. Let the plateau's maximal value be $DF_{max}(k) = \text{Max}\{DF(k, t) \mid t \in [0, T]\}$. Let r($k$) equal the total number of such maximal regions. Plateaus of value $DF_{max}(k)$ are defined by adjacent points $M^k_r = [s^k_r , e^k_r]$, $r = 1,\ldots, r(k)$, where r is the rth maximal interval ordered from the left. Note that s^k_r and e^k_r represent times of the departure and arrival flights from and to terminal k, respectively. The one exception occurs when the DF reaches its maximum value at the end of the horizon, in which case $M^k_{r(k)}$ has a departure not followed by an arrival, and $e^k_{r(k)} = T$. Hollow and their regions are indicated as: $H^k_1 = [0, s^k_1]$, $H^k_2 = [e^k_1, s^k_2]$,…, $H^k_{r(k)+1} = [e^k_{r(k)}, T]$. The schedule horizon of DF(k, t) may now be partitioned into a sequence of alternating hollow and maximal intervals, *i.e.,* ($H^k_1, M^k_1, H^k_2, \ldots, M^k_r , H^k_r ,\ldots, M^k_{r(k)}, H^k_{r(k)+1}$). An example of a FS and its corresponding set of DFs is shown in Figure 1.

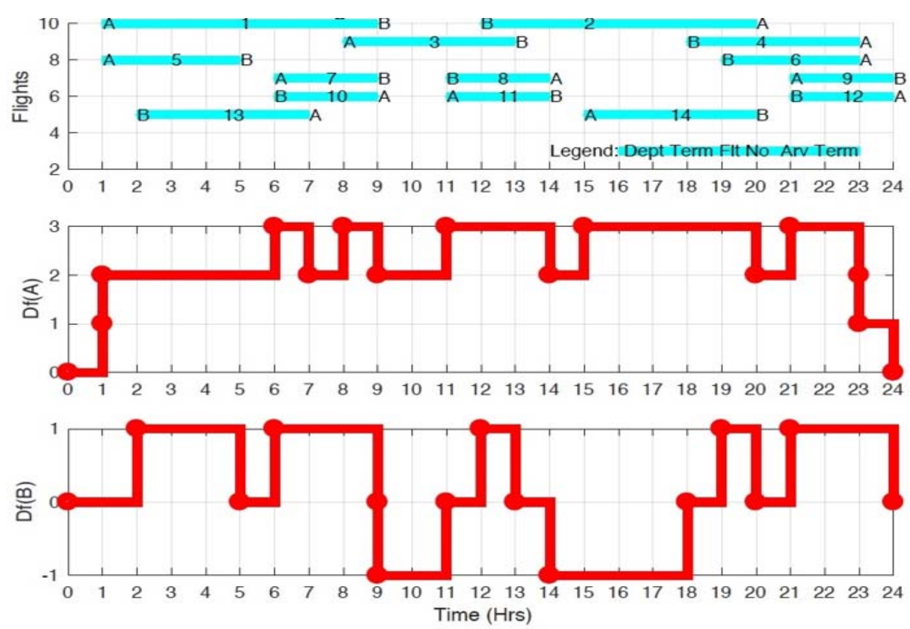

FIGURE 1. EXAMPLE OF A 14 FLIGHT SCHEDULE AND ITS DEFICIT FUNCTIONS.

An important property of DFs is that they allow a simple determination of the minimum fleet size, *i.e.,* the smallest number of aircraft required to service the FS. This is the fleet size that we wish to maintain throughout the search for a set of feasible aircraft routings. Linis and Maksim (1967) found that m, the minimum number of aircraft required to service a FS, equals the sum of the maximum values of all q DFs, *i.e.*,

$$m = \sum_{k=1}^{q} \mathrm{DF}_{\max}(k) = \sum_{k=1}^{q} \mathrm{Max}\{\mathrm{DF}(k, t) \mid t \in [0, \mathrm{T}]\} \quad (1)$$

In addition, the DF(k, t) provide the number of aircraft that start and end their routes at terminal k (not necessarily the same aircraft). For the example in Figure 1, we see from the $\mathrm{DF}_{\max}$ values that the minimum fleet size m is 4 and the number of aircraft starting from terminals A and B are 3 and 1, respectively.

3.2. Hollow Analysis and Feasible Flight Joinings

Hollows are key regions of DFs to look for feasible joinings between arrival and departure flights which form aircraft routes. In order for a flight j to be joined to a previous flight i, there needs to be a feasible joining. Two flights i and j may be feasibly joined by the same aircraft only if the following precedence relation R is satisfied.

$$\mathrm{R}: i \prec j \ni t^i_{\mathrm{a}} \leq t^j_{\mathrm{d}} \text{ and } k^j_{\mathrm{d}} = k^i_{\mathrm{a}} \quad (2)$$

Each hollow may be represented as a bipartite graph whose nodes belong to two disjoint sets I and J. Nodes in I and J are comprised of arrival and departure flight numbers, respectively. Arcs directed from I to J represent feasible flight connections.

The key operation in forming aircraft routes is "feasible flight joinings." Gertsbakh and Gurevich (1977) have shown an arriving flight in any hollow must be connected to a departing flight in the same hollow in order to achieve a set of aircraft routes that maintain the minimal fleet size. Conversely, connecting an arrival flight in a hollow to a departing flight in any other hollow will result in the set of routes exceeding the minimal fleet size. In addition, the following connection rule allows the construction of m routes such that each flight appears in exactly one route.

Expanded Flight Joining Rule

Flights i and j can be joined if and only if:

(i) Arrival Time Flight i (with turn around) ≤ Departure Time Flight j,
(ii) Arrival Terminal Flight i = Departure Terminal Flight j, and
(iii) Flights i and j appear in the same hollow of DF(Term)

In other words, flights i and j in any hollow H^k_r may be feasibly joined only if $k^j_{\mathrm{d}} = k^i_{\mathrm{a}} = k$ and $e^k_r < t^i_{\mathrm{a}}$, $t^j_{\mathrm{d}} < s^k_{r+1} \ni \mathrm{H}^k_r = [e^k_r, s^k_{r+1}]$.

IV. FORMULATION OF THE AMRP USING HOLLOW GRAPHS

4.1. Notation

The following notation pertains to a FS, its representation as a connection graph G, and our AMRP formulation. We use the terms commodity and terminal interchangeably.

I = $\{i: i = 1, \ldots, n\}$, the set of required flight legs.
FS = the set of all flight legs $\{i \in \mathrm{I}\}$, a flight schedule.
K = $\{k: k = 1, \ldots, q\}$, the set of terminals (airports) or commodities.
m = the minimum number of aircraft needed to service a FS (minimum fleet size).
t^i_{d}, t^i_{a} = flight i's departure and arrival times, respectively.
k^i_{d}, k^i_{a}, = flight i's departure and arrival terminals, respectively.
(t^i_{d} , t^i_{a}, k^i_{d}, k^i_{a}) = a flight leg i ($k^i_{\mathrm{d}} \neq k^i_{\mathrm{a}}$).
[k^{s} - i^1- i^2,-….,- i^{w} - k^{e}] = a route for a sequence of w flights with start (end) terminals k^{s} (k^{e}).
s^k = origin node for terminal k.
t^k = destination node for terminal k.
S = set of supply (origin) nodes: S = $\{\mathrm{s}^k \mid k \in \mathrm{K}\}$, $|\mathrm{S}| = q$.
T = set of demand (destination) nodes: T = $\{\mathrm{t}^k \mid k \in \mathrm{K}\}$, $|\mathrm{T}| = q$.
$\mathrm{N} = \mathrm{I} \cup \mathrm{S} \cup \mathrm{T}$

G = [N, A] is a directed acyclic graph representing a connection network of a FS, where N and A are the node and arc sets, respectively.
A predecessor/successor of node i is a node j connected to i by a directed arc (j i)/(i j).
B(i) = set of predecessor nodes of node i.
A(i) = set of successor nodes of node i.
S^k = an integer equal to the supply of commodity k at node s^k.
T^k = an integer equal to the demand of commodity k at node t^k.
M = set of maintenance terminals, $M \subset K$.

From the set of DFs, we can obtain m, the minimum fleet size (number of aircraft routes), the total number of routes, S^k, starting (supply) and, T^k, ending (demand) at each terminal k.

4.2. Hollow Graphs

We formulate the AMRP as a dual-objective integer multicommodity flow problem (MCFP) which rests upon an underlying flow graph. In aircraft routing formulations such a graph is called *a connection graph*, where flights are represented by nodes and feasible connections (joinings) between pairs of flights as directed arcs. The size of a connection graph varies according to the assumptions made by various researchers. We introduce a compact version of the common connection graph called a *hollow graph*.

Each hollow may be represented as a bipartite graph, whose nodes belong to two disjoint sets I and J. Nodes in I are comprised of arrival flight numbers while nodes in J are comprised of departure flight numbers. Arcs directed from I to J represent feasible flight connections. These bipartite graphs are coalesced as subgraphs to form a connection graph which we denote as a *hollow graph* G = [N, A]. Each node of G appears in two bipartite subgraphs, one representing the flight's arrival and the other its departure. A *gluing operation* is used to collapse these two nodes so that a flight is represented by a single node. This results in each node appearing in two overlapping binary subgraphs, once in the left set of nodes and once on the right set of nodes. The exception is for departing and arrival flights that appear in special start and end hollows and are connected directly to source and sink nodes. These sources and sink nodes are dummy terminal nodes, added to represent the counterparts of these flights. For example, in Figure 1 the three flight arrivals in the last hollow of terminal A will be connected to a dummy terminal node A. The arc set A contains all arcs in the bipartite subgraphs plus the arcs emanating and entering the supply and demand nodes S and T. This results in the size of the node set N being $n + 2q$. A hollow graph is a minimum fleet size sustainable graph. Any additional arcs between flight nodes, although representing feasible connections, may lead to solutions requiring additional aircraft.

4.3. Formulating the AMRP as a Dual Objective Integer MCFP

Two objective functions are logical candidates for the model. One is to maximize the number of routes that are cyclic and the other is to maximize the number of maintenance feasible routes. As the problem is formulated for a daily schedule, all routes may not be cyclic or maintenance feasible. Thus, increasing the number of routes that are cyclic may result in a decrease in the number that are maintenance feasible, and vice-versa. It is then natural to identify the Pareto frontier. Since some routes may end up not being cyclic, it is possible for a route to reach a demand node of a different commodity. For example, a unit flow from s^A need not reach t^A, but may arrive at another demand node $t^k \neq t^A$. Thus, maximizing the flows from s^k to t^k is equivalent to the first objective of maximizing the number of cyclic routes. In what follows we use the terms *balanced* for cyclic routes and *unbalanced* for noncyclic routes. First, let R be the set of all triples $\{ijk\}$ representing the possibility of commodity k flowing on arc (*ij*). Our model uses a binary decision variable for each element of R.

$$x_{ij}^{k} \in \{0,1\}, \ \forall \{ijk\} \in R \qquad (3)$$

In this context, a decision variable is 1 if a route starts at terminal *k* and includes flights *i* and *j* in sequence, and 0 otherwise. Two sets of auxiliary binary variables are employed to compute the total number of balanced routes and maintenance feasible routes, respectively. In particular, the variable b_{ij}^{k} equals one only if a route starts at terminal k, and passes through flight *i* immediately before ending at terminal *k*, while f_{ij}^{k} equals one only if a route passes through flight *i* immediately after or before a maintenance terminal, *i.e.*, if either $k \in$ M or $\mathrm{t}^{j} \in$ M.

The model has six constraints sets, in addition to those defining the decision variables.

$$\sum_{j \in \mathrm{A}(\mathrm{s}^k)} x_{\mathrm{s}^k j}^{k} = \mathrm{S}^k, \ \ \forall k \in \mathrm{K} \qquad (4)$$

$$\sum_{w \in \mathrm{K}} \sum_{i \in \mathrm{B}(\mathrm{t}^k)} x_{i\mathrm{t}^k}^{w} = \mathrm{T}^k, \ \ \forall k \in \mathrm{K} \qquad (5)$$

$$\sum_{k \in \mathrm{K}} \sum_{i \in \mathrm{B}(j)} x_{ij}^{k} = 1, \ \ \forall j \in \mathrm{I} \qquad (6)$$

$$\sum_{i \in \mathrm{B}(j)} x_{ij}^{k} = \sum_{i \in \mathrm{A}(j)} x_{ji}^{k}, \ \ \forall j \in \mathrm{I} \text{ and } k \in \mathrm{K} \qquad (7)$$

$$b_{ij}^{k} = (k == j)\, x_{ij}^{k}, \ \ \forall \{ijk\} \in R \ni j \in \mathrm{K} \text{ and } i \in \mathrm{B}(\mathrm{t}^j) \qquad (8)$$

$$f_{ij}^{k} = (k \in M \ || \ j \in M)\, x_{ij}^{k}, \ \ \forall \{ijk\} \in R \ni j \in \mathrm{K} \text{ and } i \in \mathrm{B}(\mathrm{t}^j) \qquad (9)$$

$$x_{ij}^{k} \in \{0,1\}, \ \ \forall \{ijk\} \in R \qquad (10)$$

$$b_{id}^{k},\ f_{id}^{k},\ \in \{0,1\}, \ \ \forall \{ijk\} \in R \ni j \in \mathrm{K} \text{ and } i \in \mathrm{B}(\mathrm{t}^j) \qquad (11)$$

Constraints (4) and (5) are commodity supply and demand constraints, respectively. Although Eq. (5) allows unbalanced routes between non-identical terminals (commodities), Eq. (8) has been added to count the number of such unbalanced routes (which may be minimized in the objective function). Constraints (6) insure the flow into each flight node is one, so that each flight lies on exactly one route. Constraints (7) are conservation of flow constraints for the flight nodes. Constraints (8) and (9) compute binary variables indicating whether or not end-of-day arcs lie on balanced and maintenance feasible routes, respectively. The logical expressions embedded within these constraints are available in many modern algebraic modeling systems, such as the one we used to formulate and solve the problem (CPLEX). Decision variable declarations are repeated in (10) and (11) for completeness. Unlike set partitioning based formulations which contain exponentially many variables, this formulation uses a polynomial number of variables and constraints, where the number of variables is ~ q|A| and the number of constraints is $2q+n+nq+m = n(1+q)+2q+m \sim qn+m$.

We consider the two objectives shown in (12) and (13).

$$\text{Max TB} = \sum_{k \in \mathrm{K}} \sum_{i \in \mathrm{B}(\mathrm{t}^k)} b_{i\mathrm{t}^k}^{k} \qquad (12)$$

$$\text{Max TF} = \sum_{k \in \mathrm{K}} \sum_{i \in \mathrm{B}(t^k)} f_{i\mathrm{t}^k}^{k} \qquad (13)$$

To find the Pareto frontier we define problem P($\mathrm{R_B}$) which maximizes TF (13), subject to constraints (4)-(11) plus an additional constraint (14), which is just (12) with a fixed value of $\mathrm{R_B}$.

$$\sum_{k \in K} \sum_{j \in B(t^k)} b_{ij}^{k} = \mathrm{R_B} \qquad (14)$$

Solving a sequence of problems $P(R_B)$ for R_B ranging from 0 to TB* generates a set of solutions that form a Pareto frontier. If the optimal solution to $P(R_B)$ is TF_B^*, then the solution to $P(R_F)$ can be denoted as the ordered pair $[R_B, TF_B^*]$. The set of such ordered pairs for all values of R_F trace out the Pareto frontier. TF* denotes the largest number of maintenance feasible routes found for $P(R_B)$ after solving across all possible values of R_B.

V. EXAMPLE PROBLEM WITH 30 FLIGHTS AND 4 TERMINALS

To illustrate, we use a 30 flight, 4 terminal example problem whose FS is given in the Appendix. The corresponding DFs are shown in Figure 2 below.

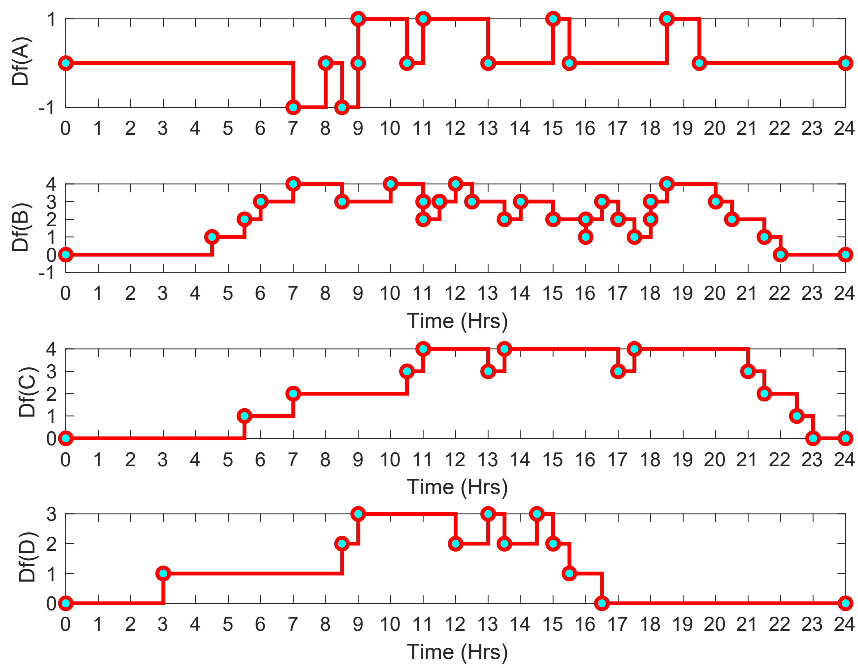

FIGURE 2. DFS FOR THE 30 FLIGHT, 4 TERMINAL PROBLEM.

For this problem the number of aircraft starting their routes at each terminal is $DF_{max}(k) = S^k = T^k = 1, 4, 4, 3$ for k = A, B, C, D, respectively, so the minimum fleet size $m = \sum_{k\in K}(S^k) = \sum_{k\in K}(T^k) = 12$. The associated hollow graph G appears below in Figure 3, where the nodes labeled 1 to 30 correspond to the 30 flights given in the FS. In all there are 10 hollows in G represented by their bipartite subgraphs. In addition, there are 4 source nodes and 4 sink nodes labeled by s^k and t^k, respectively. The arc set of size 71 is comprised of flight connection arcs and commodity supply and demand arcs. Terminals B and C are assumed to be maintenance terminals so the 8 routes starting at nodes s^B and s^C are maintenance feasible.

We solved this problem using the well-known mathematical programming software package ILOG CPLEX Optimization Studio, available from IBM (2019). Solving $P(R_B)$ for R_B = 3, 4, …, 7 generates the Pareto frontier shown in Figure 4. (Problems outside this range of R_B values were infeasible.) The Pareto graph illustrates the trade-off between balanced and maintenance feasible routes. For example, the maximum number of maintenance feasible solutions of 12 can only be achieved when the second objective does not exceed 4 balanced routes. Details of the routes in the maximum maintenance feasible solution are shown in Table 1 below.

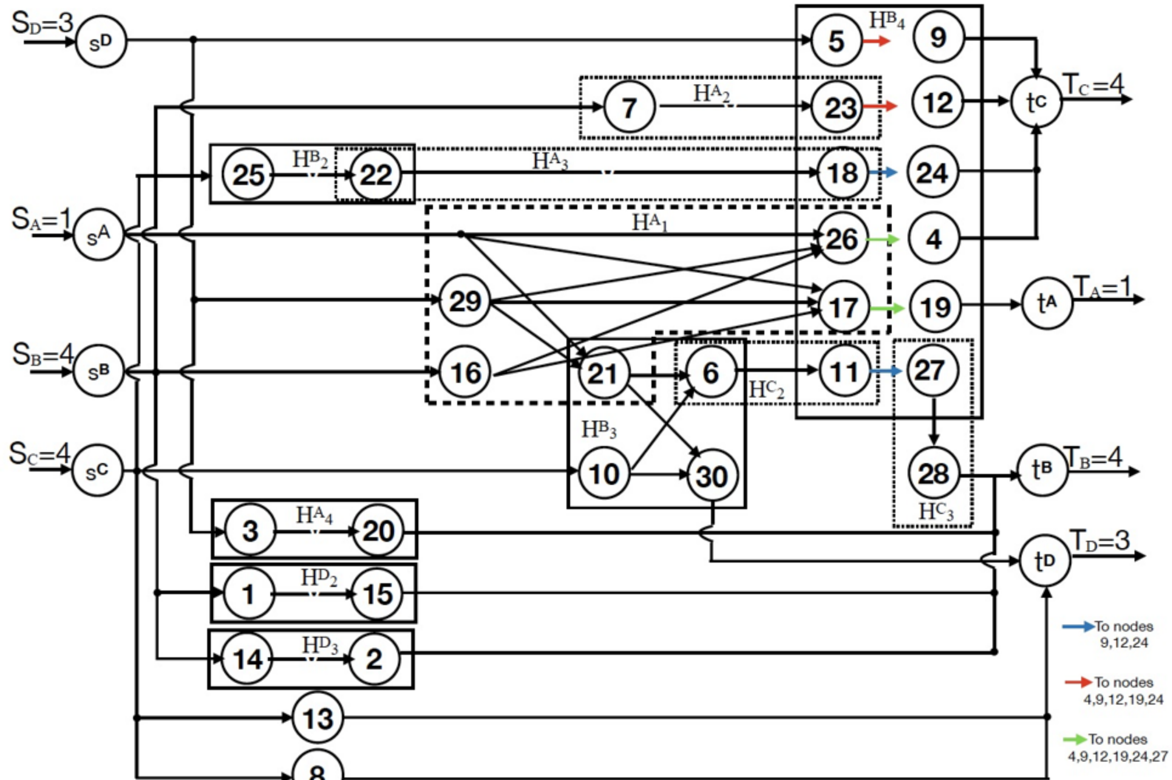

FIGURE 3. HOLLOW GRAPH FOR THE 30 FLIGHT, 4 TERMINAL PROBLEM.

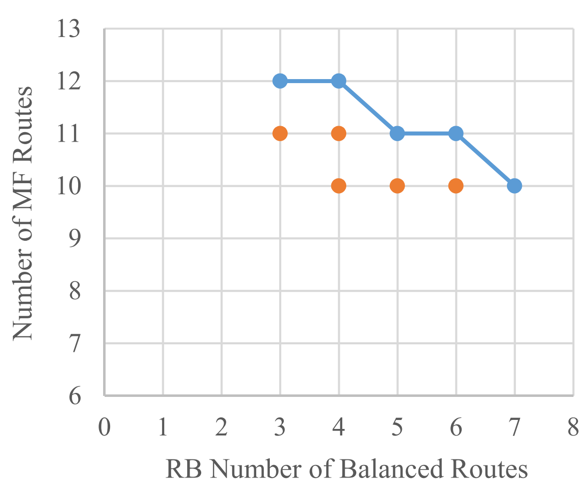

FIGURE 4. PARETO FRONTIER FOR DUAL OBJECTIVES OF EXAMPLE PROBLEM.

TABLE 1. MAX MF PARETO SOLUTION FOR THE EXAMPLE PROBLEM

	Route	Bal?	MF?		Route	Bal?	MF?
R1	A-17-4-C	No	Yes	R7	C-10-30-D	No	Yes
R2	B-1-15-B	Yes	Yes	R8	C-13-D	No	Yes
R3	B-7-23-19-A	No	Yes	R9	C-8-D	No	Yes
R4	B-14-2-B	Yes	Yes	R10	D-3-20-B	No	Yes
R5	B-16-26-27-28-B	Yes	Yes	R11	D-5-24-C	No	Yes
R6	C-25-22-18-9-C	Yes	Yes	R12	D-29-21-6-11-12-C	No	Yes
					Total	TB=4	TF=12

To obtain an "all balanced" solution, unbalanced routes must be converted into balanced routes. The complete details of this procedure appear in Stern and Gertsbakh (2019).

VI. PERFORMANCE EVALUATION

Our method and formulation cannot be directly evaluated against other formulations that have appeared in the literature, primarily because the assumptions behind the approaches are inconsistent and the lack of documented details. Therefore, our performance evaluation will be based on the types of connection graphs and the assumption on the number of commodities. Two types of evolution are made here, one based on problem size and the other on run time performance. The first is measured by size of the underlying network, which directly affects the problem size, and the second by run time ticks. A tick is a deterministic unit of computational work whose correspondence to clock time varies by platform but which is consistent for a given platform carrying the same load.

6.1. Comparing Our Formulation to Previous AMRP Approaches

Compared to the set covering approach we do not need to generate a large sample of routes. Most other formulations assume the size of the aircraft fleet is equal to the existing fleet of the airline. By contrast, we find the minimum fleet size aircraft fleet needed to service the existing FS. In addition, our formulation allows for dual objectives: one maximizing the number of cyclic routes and another maximizing the number of maintenance feasible routes. We also construct a reduced flight connection network compared to those used in other formulations. Most formulations assume each plane in the fleet is a commodity, inducing many more feasible connection arcs in the underlying connection network, as well as replicating the set of commodity flow variables. Our formulation, instead, uses only a number of commodities equal to the number of terminals, which is generally much lower than the number of planes.

Because we cannot directly compare our formulation with others we will focus on the advantage of our hollow connection graph and the number of commodities used in our formulation compared to others in the literature. The number of connection arcs in our formulation is the minimum required to preserve the minimum fleet size. Others determine the number of flight connections by considering candidate connections based on only the first two conditions of our expanded flight joining rule. Two common methods are to (1) allow a connection from a flight node to the *w* nearest feasible flight nodes, and (2) allow connections to *all* flight nodes that satisfy the first two feasibility conditions. Using these methods, we conduct a comparative connection graph experiment for 3 different connection graph cases. Moreover, in the traditional MCFP, each route from a source to a sink is a single commodity. In our formulation all routes emanating from a single terminal are grouped as a single commodity, significantly reducing problem size since, in most real aircraft situations, the number of aircraft routes is much larger than the number of aircraft terminals.

6.2. Evaluation Based on Connection Graphs

In his doctoral thesis Gronkvist (2005) describes a number of connection network reduction techniques, describing how the connection network can be preprocessed to reduce its size. The most applicable to ours is the *w*-FIFO heuristic network filtering method which allows connections to the first *w* feasible departing fights. However, such heuristics do not filter out all unnecessary connections. Our hollow graph connection graph, by contrast, contains only the absolute minimum number of connections arcs. We compare 3 types of connection graphs for the 4-commodity case:

1) **Hollow Graph**: Feasible connections in the hollows of all DFs according the Expanded Flight Joining Rule are considered.
2) **4 Nearest Connections (4-FIFO)**: For a given flight arrival at a terminal, the 4 nearest feasible departures are considered.
3) **All Feasible Connections**: For a given flight arrival at a terminal, all feasible "downstream" departures are considered (which may include out-of-hollow departures).

In all three cases, connections directly from S^k to T^k are not counted. Table 2 depicts the number of connection arcs in each case for the 30 flight, 4 terminal example problem.

TABLE 2. COMPARISON OF THE NUMBER OF ARCS FOR 3 TYPES OF CONNECTION GRAPHS (USING 4 COMMODITIES)

Deficit Function	Hollow Graph	4 Nearest Connections	All Feasible Connections
DF(A)	12	22	29
DF(B)	41	44	78
DF(C)	10	13	15
DF(D)	8	12	13
Total	71	91	135

TABLE 3. COMPARISON OF NETWORK SIZES FOR 4 AND 12 COMMODITIES CASES

4 Commodities Case	Hollow Graph[a]	4 Nearest Connections	All Feasible Connections
Commodity Flow Variables	127	165	253
Constraints	247	313	388
Run Time (ticks)	1.72	2.10	3.27
12 Commodities Case			
Commodity Flow Variables	517	768	1122
Constraints	1106	1691	2171
Run Time (ticks)	16.57	18.03	32.48

[a] The network size is comprised of the hollow graph plus the arcs and nodes from and to the commodity sources and sinks.

6.3 Evaluation Based on Number of Commodities

While many authors consider each aircraft as a commodity, our formulation treats all aircraft flowing out of a given terminal as a single commodity. Since the number of terminals is normally much smaller than the number of aircraft in a given fleet, our approach considerably reduces both the size of the underlying MCF graph and the number of commodity flow variables.

6.4 Discussion of Results

Table 2 shows that our hollow graph formulation provides an appreciable decrease in the number of arcs of the connection graph ranging from 22 to 47%. Consequently, there are corresponding reductions in model size (number of variables and constraints) and run time, as seen in Table 3. In particular, in the four commodities case, the hollow graph connection network reduces the number of commodity flow variables by 50% and 23%, respectively, over that required by the all-feasible and four nearest connections networks; in the 12 commodities case, the reductions were 54% and 33%, respectively. These improvements lead to faster model run times. Table 3 indicates that using the hollow connection graph as opposed to the all-feasible connection graph yields a 47% speed up when both formulations use four commodities. Furthermore, comparing our formulation based on the hollow graph connection network with four commodities to that based on the all-feasible connection graph with 12 commodities shows an 89% decrease in the number of commodity flow variables (from 1122 to 127) and a 95% drop in solution time over (from 32.48 to 1.72 ticks).

VII. CONCLUSION AND FUTURE WORK

This article has provided a new formulation of the AMRP for a repeated daily flight schedule with a homogeneous fleet of aircraft. We believe it is the first formulation to integrate the minimum fleet size found from deficit function theory into the AMRP. Using a deficit function representation of FS, with regions between maximal values called hollows, we extended the classical flight joining rule so that the number of arcs in the connection graph is appreciably reduced. This formed the basis of a new type of connection graph called a hollow graph. Moreover, our formulation ensures that the plane count remains equal to the minimum fleet size. We compared the size and run times of our formulation to other formulations using a variety of connection graphs and number of commodities, and showed that our approach reduced problem size by as much as 89% and solution time by up to 95% for a 30-flight, 4-terminal example.

Our approach has several advantages over the two main formulations seen in the literature, the set covering approach and the multi-period formulation. First, those approaches solve the AMRP without integrating the minimum fleet size condition, yielding solutions that require more aircraft than are necessary, up to an amount equal to the number of aircraft already owned by the airline. Second, other papers fail to consider the trade-off between maintenance feasible routes and cyclic routes. Classically, each route in the solution should be maintenance feasible, meaning that each aircraft should visit a maintenance operation. We added the important condition that routes should be cyclic. Maximizing the number of cyclic routes is crucial in practice as it allows crews and aircraft to return to their home bases, reducing the cost of overnight stays. Thus, we expanded AMRP by considering it as a dual-objective MCF ILP problem. Furthermore, from a mathematical programming perspective, the MCF formulation that we use has only a polynomial number of variables whereas the set covering approach has an exponential number of variables.

Finally, we were surprised to find no literature analyzing the optimization of one of the main capital resources of an airline, i.e., the number of planes in the fleet. All scheduling analysis assumed the current number of planes as a given. The advantages of finding the minimum fleet size is that, once it is found, airlines can reduce or increase their existing fleet accordingly. Placing our work in the context of others, the salient difference is that we introduce a procedure to find the maximal number of daily (single-period) balanced and maintenance feasible routes, while ensuring a solution that uses the minimum number of planes. While we developed this methodology for the single-period AMRP problem, we intend to use it as a springboard to solve problems with a multi-period FS. In addition, we plan to include constraints on the available maintenance capacity of each terminal.

REFERENCES

Barnhart, C., Boland, N.L., Clarke, L.W., Johnson, E.L., Nemhauser, G.L. and Shenoi, R.G., "Flight string models for aircraft fleeting and routing," *Transportation Science*, 32(3), 1998, 208-220.

Cordeau, J.F., Stojkovi´c, G, Soumis, F., Desrosiers J., "Benders decomposition for simultaneous aircraft routing and crew scheduling," *Transportation Science*, 35, 2001, 375–388.

Gertsbakh I., and Gurevich, Y., "Constructing an optimal fleet for a transportation schedule," *Transportation Science*, 11(1), 1977, 20–36.

Gertsbakh I., and Gurevich, Y., "Homogeneous optimal fleet," *Transportation Research Part B Methodology,* 16(6), 1982, 459-470.

Gertsbakh I., and Stern, H.I., "Minimal resources for fixed and variable job schedules," *Operations Research*, 26, 1978, 68-85.

Gertsbakh I., and Stern, H.I., "Using DFs for crew planning in aviation," *Transport and Telecommunication,* 18(4), 2017, 289-296.

Gronkvist, M., "The tail assignment problem," Doctoral thesis, Chalmers University of Technology, 2005, https://research.chalmers.se/en/publication/6207 (accessed September 27, 2019).

Haouari, M., Shao S., Sherali, H.D., "A lifted compact formulation for the daily aircraft maintenance routing problem," *Transportation Science*, 47(4), 2013, 508-525.

Khaled, O., Minoux, M., Mousseau, V., Michel, S., Ceugniet, X., "A compact optimization model problem for the tail assignment problem," *European Journal of Operational Research*, 264(2), 2018, 548-557.

Lacasse-Guay, E., Desaulniers, G., and Soumis, F., "Aircraft routing under different business processes," *Journal of Air Transport Management*, 16, 2010, 258-263. 10.1016/j.jairtraman.2010.02.001.

Linis, V.K., and Maksim, M.S., "On problem of constructing routes (in Russian)," *Proc Civil Aviation Institute*, Riga 102, 1967, 36–45.

Liu, T., and Ceder, A., "Deficit function related to public transport: 50-year retrospective, new developments, and prospects." *Transportation Research Part B: Methodological* 100, 2017, 1-19. doi: 10.1016/j.trb.2017.01.015

Maher, S.J., Desaulniers, G., and Soumis, F., "Recoverable robust single day aircraft maintenance routing problem," *Computers and Operations Research*, 51, 2014, 130-145.

DOI=http://dx.doi.org/10.1016/j.cor.2014.03.007

IBM, "IBM ILOG CPLEX Optimization Studio," https://www.ibm.com/products/ilog-cplex-optimization-studio?mhsrc=ibmsearch_a&mhq=CPLEX (accessed September 27, 2019).

Stern, H.I. and Gertsbakh, I.B., "Using deficit functions for aircraft fleet routing," *Operations Research Perspectives*, 6, 2019, https://doi.org/10.1016/j.orp.2019.100104 (accessed September 27, 2019).

Xu, Y., Wandelt, S. and Sun, X., "Stochastic tail assignment under recovery," *Thirteenth USA/Europe Air Traffic Management Research and Development Seminar*, June 2019.

APPENDIX

Flight Schedule for the 30 flight, 4 terminal example.

Flight No.	Depart Time	Arrival Time	Depart Term	Arrival Term	Flight No.	Depart Time	Arrival Time	Depart Term	Arrival Term
1	4.50	12.00	2	4	16	5.50	8.50	2	1
2	14.50	20.50	4	2	17	9.00	12.50	1	2
3	8.50	15.50	4	1	18	15.00	17.50	1	2
4	16.50	21.00	2	3	19	16.00	19.50	2	1
5	9.00	16.00	4	2	20	18.50	22.00	1	2
6	11.50	13.00	2	3	21	8.00	11.00	1	2
7	6.00	10.50	2	1	22	10.00	13.00	2	1
8	11.00	16.50	3	4	23	11.00	15.00	1	2
9	18.00	21.50	2	3	24	18.50	22.50	2	3
10	7.00	11.00	3	2	25	5.50	8.50	3	2
11	13.50	17.00	3	2	26	9.00	13.50	1	2
12	18.00	23.00	2	3	27	14.00	17.00	2	3
13	10.50	15.50	3	4	28	17.50	21.50	3	2
14	7.00	13.50	2	4	29	3.00	7.00	4	1
15	13.00	20.00	4	2	30	12.00	15.00	2	4