A Fuzzy Non-Linear Programming Economic Order Quantity Model with Demand Dependent Unit Cost of Production under Two Constraints

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A fuzzy NLP EOQ model is developed with demand dependent unit cost of production and dynamic setup cost under limited capital investment and limited storage capacity. Here fuzziness is introduced in objective function, capital system and storage area. It is solved by fuzzy non-linear technique for linear membership functions. This paper allows the modification of the Single item EOQ model in presence of fuzzy decision making process for demand dependent unit cost of production in the presence of imprecisely estimated parameters. The model is developed for the problem by employing different modeling approaches over an infinite planning horizon. It incorporates all concepts of a fuzzy arithmetic approach, the quantity ordered and the demand per unit compares both fuzzy non linear and other models. Computational algorithm using the LINGO 13.0 version software is developed to find the optimal solution and the diagrammatical representations can be obtained by MATLAB 7.8.0. (R2009a) version software. Investigation of the properties of an optimal solution allows developing an algorithm whose validity is illustrated through an example problem. Sensitivity analysis of the optimal solution is also studied with respect to changes in different parameter values and to draw managerial insights. By a comparative study of a numerical example, it demonstrates the efficiency of the available formulae in the literature to highlight the optimality of the solution technique satisfying two constraints.
Keywords: Fuzzy, NLP, EOQ, Budget Constraint, Storage Capacity

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I. INTRODUCTION

Since its formulation in 1915, the square root formula for the economic order quantity (EOQ) was used in the inventory literature for a pretty long time. Ever since its introduction in the second decade of the past century, the EOQ model has been the subject of extensive investigations and extensions by academicians. Although the EOQ formula has been widely used and accepted by many industries, some practitioners have questioned its practical application. For several years, classical EOQ problems with different variations were solved by many researchers and had be separated in reference books and survey papers e.g. Taha [5], Urgeletti [3]. Recently, for a single product with demand related to unit price Cheng [21] and for multi products with several constraints. His treatments are fully analytical and much computational efforts were needed there to get the optimal solution.

Operations Research (OR) was first coined in 1940 by Mcclosky and Trefther in a small town, Bowdsey, in the UK. During the Second World War, this OR mathematics was used in a wider sense to solve the complex executive strategic and tactical problems of military teams. Since then the subject has been enlarged in importance in the field of Economics, Management Sciences, Public Administration, Behavioral Science, Social Work Commerce Engineering and different branches of Mathematics etc. But various Paradigmatic changes in science and mathematics concern the concept of uncertainty. In Science, this change has been manifested by a gradual transition from the traditional view, which insists that uncertainty is undesirable and should be avoided by all possible means. According to the traditional view, science should strive for certainty in all its manifestations; hence uncertainty is regarded as unscientific. According to the modern view, uncertainty is considered essential to science; it is not any an unavoidable plague but has; in fact, a great utility. But to tackle non-random uncertainty no other mathematics was developed other than fuzzy set theory and showed the intention to accommodate uncertainty in the presence of random variables. Following Zadeh [11], significant contributions in this direction have been applied in many fields including production related areas. Consequently investment in introducing fuzzy is the key to avoid uncertain decision space. Many studies have modified inventory policies by considering the issues of nonrandom uncertain and fuzzy based EOQ models. Vujosevic et al. [14] presented a theoretical EOQ formula when inventory cost is fuzzy. Lee et al. [7] studied an inventory model for fuzzy demand quantity and fuzzy production quantity. Tripathy et al. [16, 18, 19] introduced the concept and developed the framework for investing fuzzy in holding cost and setup cost in EOQ model. Tripathy et al. [17] suggested improvements to production systems by employing entropy in the fuzzy model. Pattnaik [12] extends by considering stock dependent demand rate with entropy factor in the crisp non linear EOQ model.

Sommer [2] applied fuzzy dynamic programming to an inventory and production scheduling problem in which the management wishes to fulfill a contract for providing a product and then withdraw from the market. Kacprzyk et al. [9] introduced the determination of optimal of firms from a global view point of top management in a fuzzy environment with fuzzy constraints improved on reappointments and a fuzzy goal for preferable inventory levels to be attained. Park [10] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Here, inventory costs were represented by trapezoidal fuzzy numbers (TrFN) and the EOQ model was transformed to a fuzzy optimization problem. Tripathy et al. [15] introduce the concept of promotional effort for deteriorating items in crisp instantaneous EOQ model. Pattnaik [13] extends concept of promotional effort for deteriorating items in fuzzy instantaneous replenishment model.

But Roy et al. [22], Roy et al. [23] have considered the space constraint with the objective goal in fuzzy environment and attacked the fuzzy optimization problem directly using either fuzzy non-linear or fuzzy geometric programming technique similarly Lee et al. [7] and Vujosevic et al. [14] have applied fuzzy arithmetic approach in EOQ model without constraints.

TABLE 1. SUMMARY OF THE RELATED RESEARCHES

Authors	Demand	Setup cost	Holding cost	Unit cost of production	Constraints	Planning horizon	Structure of the Model	Model class
Vujosevic et al. (1996)	Constant	Constant	$\frac{\tilde{c}_h c_p Q}{2 \times 100}$	Constant	No	Finite	Fuzzy	Defuzzification
Tripathy et al. (2009)	Constant	Constant	$\frac{Hr^2q^2}{2\lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. (2011)	Constant	Constant	$\frac{H\lambda q^2}{2r^2}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. (2011)	Constant	Constant	$\frac{Hq^2}{2r^2\lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Roy et al. (1995)	Constant	Variable	$\frac{1}{2}C_1 q$	No	Space	Infinite	Fuzzy	NLP
Roy et al. (1997)	Constant	Variable	$\frac{1}{2}C_1 q$	Demand	Space	Infinite	Fuzzy	NLP, GPP
Present paper (2015)	Constant	Variable	$\frac{1}{2 \times 100} C_1 K D^{-\beta} q$	Demand	Budget and Storage Capacities	Infinite	Fuzzy	NLP

In this paper a single item EOQ model is developed where unit price varies inversely with demand and setup cost increases with the increase of production. In company or industry, total expenditure for production and storage area are normally limited but imprecise, uncertain, non-specificity, inconsistency vagueness and flexible. These are defined within some ranges. However, the no stochastic and ill formed inventory models can be realistically represented in the fuzzy environment. The problem is reduced to a fuzzy optimization problem associating fuzziness with the storage area and total expenditure. The optimum order quantity is evaluated by both fuzzy non linear programming (FNLP) method and the results are obtained for linear membership functions. The model is illustrated with numerical example and with the variation in tolerance limits for both shortage area and total expenditure. A sensitivity analysis is presented. The numerical results for fuzzy and crisp models are compared. The remainder of this paper is organized as follows. In section 2, assumptions and notations are provided for the development of the model and the mathematical model is developed. In section 3, mathematical analysis of fuzzy non linear programming (FNLP) is formulated. The solution of the FNLP inventory is derived in section 4. The numerical example is presented to illustrate the development of the model in section 5. The sensitivity analysis is carried out in section 6 to observe the changes in parameters in the optimal solution. Finally section 7 deals with the summary and the concluding remarks.

II. MATHEMATICAL MODEL

A single item inventory model with demand dependent unit price and variable setup cost under storage constraint is formulated as

Min C (D,q)
$= C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2\times 100}C_1KD^{-\beta}q$
s.t.$\frac{1}{2}uq \leq U$
$Aq \leq B$
$\forall$ D, q > 0

(1)

Where,
q = number of order quantity,
D = demand per unit time
C_1 = holding cost per item per unit time.
C_3 = Setup cost = C03 q^{ν},
(C_{03} (> 0) and ν (0< ν< 1) are constants)

p = Unit production cost = $KD^{-\beta}$, K (> 0) and β (> 1) are constants. Here lead time is zero, no back order is permitted and replenishment rate is infinite. U, u, A and B are nonnegative real numbers, U is the capital investment goal and B is the space constraint goal. The above model in a fuzzy environment is

$\widetilde{Min}$ C (D,q)
$= C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2\times 100}C_1KD^{-\beta}q$
s. t.$\frac{1}{2}uq \leq \tilde{U}$
$Aq \leq \tilde{B}$
$\forall$ D, q > 0

(A wavy bar (~) represents fuzzification of the parameters).

(2)

III. MATHEMATICAL ANALYSIS OF FUZZY NON LINEAR PROGRAMMING (FNLP)

A fuzzy non linear programming problem with fuzzy resources and objective are defined as

$\widetilde{Min} g_0$(x)
s.t. $g_i(x) \leq \tilde{b}_i$ i=1, 2, 3, ……m
$g_j(x) \leq \tilde{u}_j$ j=1,2.3, ……..m

(3)

In fuzzy set theory, the fuzzy objective and fuzzy resources are obtained by their membership functions, which may be linear or nonlinear. Here μ_0 and μ_i (i = 1, 2 ...., m) are assumed to be non increasing continuous linear membership functions for objective and resources respectively such as

$$\mu_i(g_i(x)) = \begin{cases} 1 \ \ if\ g_i(x) < b_i, \\ 1 - \frac{g_i(x)-b_i}{P_i}\ if\ b_i \leq g_i(x) \leq b_i + P_{i,} \\ 0\ if\ g_i > b_i + P_i, \end{cases}$$
i = 0, 1, 2, ..., m.

$$\mu_j(g_j(x)) = \begin{cases} 1 \ \ if\ g_j(x) < u_i, \\ 1 - \frac{g_j(x)-b_j}{P_j}\ if\ u_j \leq g_j(x) \leq u_j + P_{j,} \\ 0\ if\ g_j > b_j + P_j, \end{cases}$$
j = 0,1, 2, …….m

In this formulation, the fuzzy objective goal is b_0 and its corresponding tolerance is P_0 and for the fuzzy constraints, the goals are b_i's and their corresponding tolerances are P_i's (i = 1, 2, ..., m). To solve the problem (3), the max - min operator of Bellman et al. [20] and the approach of Zimmermann [6] are implemented.

The membership function of the decision set,μ_D (x), is μ_D (x) = min {μ_0 (x), μ_1 (x), ..., μ_m (x)},$\forall$ x ∈ X.

The min operator is used here to model the intersection of the fuzzy sets of objective and constraints. Since the decision maker wants to have a crisp decision proposal, the maximizing decision will correspond to the value of x, x_{max} that has the highest degree of membership in the decision set.

μ_D (x_{max}) $= \max_{x\geq 0}[\min\{\mu_0(x), \mu_1(x) \ldots, \mu_m(x)\}]$. It is equivalent to solving the following crisp non linear programming problem.

Max α
s.t. $\mu_0(x) \geq \alpha$
$\mu_i(x) \geq \alpha$ (i= 1, 2, ... , m)

∀ x ≥ 0, α∈ (0, 1)

(4)

A new function, i.e the Lagrangian function L (α, x,λ) is formed by introducing (m +1) Lagrangian multipliers λ = (λ_0, λ_1, ...λ_m).

L $(\alpha, x, \lambda) = \alpha - \sum_{i=0}^{m} \lambda_i(g_i(x) - b_i - (1-\alpha)P_i) - \sum_{i=0}^{m} \lambda_i(g_i(x) - u_i - (1-\alpha)P_i)$. The necessary condition of Kuhn et al. [8] for the optimal solution to this problem implies that optimal values $x_1^*, x_2^*, x_3^*, \ldots x_n^*$ *and* $\lambda_1^*, \lambda_2^*, \lambda_3^*, \ldots \lambda_n^*$ should satisfy

$$\frac{\partial L}{\partial xj} = 0$$

j = 1, 2, ... n

$$\frac{\partial L}{\partial \alpha} = 0$$

λ_i (g_i (x) - b_i - (1 - α) P_i) = 0
λ_i (g_i (x) $- u_i - (1-\alpha)P_i$) = 0,
$g_i(x) \leq b_i + (1-\alpha)P_i$,
$g_i(x) \leq u_i + (1-\alpha)P_i$,
$\lambda_i \leq 0, i = 0, 1, \ldots, m$

(5)

Moreover, Kuhn-Tucker's sufficient condition demands that the objective function for maximization and the constraints should be respectively colane and convex. In this formulation, it can be shown that both objective function and constraints satisfy the required sufficient conditions. Now, solving (5), the optimal solution for the FNLP problem is obtained.

IV. SOLUTION OF THE PROPOSED INVENTORY MODEL

The proposed inventory model depicted by equation (2)

$\widetilde{Min}$ C (D,q)
$= C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2\times 100}C_1KD^{-\beta}q$
s. t.$\frac{1}{2}uq \leq \tilde{U}$
$Aq \leq \tilde{B}$
∀ D, q > 0, reduces to following equation (4),

Max α
s.t. $C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2\times 100}C_1KD^{-\beta}q$
$\leq C_0$ + (1 - α) P_0,

$\frac{1}{2}uq \leq U + (1-\alpha)P_1$
Aq ≤ B + (1 - α)P_2,
∀ D, q > 0 &α ∈ (0, 1)

(6)

Here, the objective goal is C_0 with tolerance P_0 and the capital investment constraint goal with tolerance P_1 and space constraint goal is B with tolerance P_2. So, the corresponding Lagrangian function is

L (α, D, q, λ_1, λ_2, λ_3)
$$= \alpha - \lambda_1\left(C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2\times 100}C_1KD^{-\beta}q - C_0 - (1-\alpha)P_0\right)$$
$$-\lambda_2\left(\frac{1}{2}uq - U - (1-\alpha)P_1\right) - \lambda_3(Aq - B - (1-\alpha)P_2)$$

From Kuhn - Tucker's necessary conditions,

$$\frac{\partial L}{\partial \alpha} = 0, \frac{\partial L}{\partial D} = 0, \frac{\partial L}{\partial q} = 0, \frac{\partial L}{\partial \lambda_1} = 0, \frac{\partial L}{\partial \lambda_2} = 0,$$

$$\frac{\partial L}{\partial \lambda_3} = 0, \forall\, \lambda_1, \lambda_2, \lambda_3 \leq 0$$

$$\frac{\partial L}{\partial \alpha} = 1 - \lambda_1 P_0 - \lambda_2 P_1 - \lambda_3 P_2 \geq 0$$

$$\frac{\partial L}{\partial D} = \lambda_1 \left(C_{03} q^{\nu-1} + (1-\beta) K D^{-\beta} - \frac{1}{2 \times 100} C_1 K q \beta^{-(\beta+1)} D \right) \leq 0$$

$$\frac{\partial L}{\partial q} = \lambda_1 \left(C_{03} (\nu - 1) q^{(\nu-2)} D + \frac{1}{2 \times 100} C_1 K D^{-\beta} \right) + \frac{u}{2} \lambda_2 + A \lambda_3 \leq 0$$

$$\frac{\partial L}{\partial \lambda_1} = \left(C_{03} q^{(\nu-1)} D + K D^{1-\beta} + \frac{1}{2 \times 100} C_1 K D^{-\beta} q \right) - C_0 - (1 - \propto) P_0 \geq 0$$

$$\frac{\partial L}{\partial \lambda_2} = \frac{1}{2} u q - U - (1 - \propto) P_1 \geq 0$$

$$\frac{\partial L}{\partial \lambda_3} = A q - B - (1 - \propto) P_2 \geq 0$$

and $\alpha(1 - \lambda_1 P_0 - \lambda_2 P_1 - \lambda_3 P_2) = 0$

$$\lambda_1 D \left(C_{03} q^{(\nu-1)} + (1-\beta) K D^{-\beta} - \frac{1}{2 \times 100} C_1 K q \beta^{-(\beta+1)} D \right) = 0$$

$$\lambda_1 q \left(C_{03} (\nu - 1) q^{(\nu-2)} D + \frac{1}{2 \times 100} C_1 K D^{-\beta} \right) + \frac{u}{2} \lambda_2 q + A \lambda_3 q = 0$$

$$\lambda_1 \left(C_{03} q^{(\nu-1)} D + K D^{1-\beta} + \frac{1}{2 \times 100} C_1 K D^{-\beta} q - C_0 - (1-\alpha) P_0 \right) = 0$$

$\lambda_2 (Aq - B - (1-\alpha) P_1) = 0$, $\forall\, \alpha$, D, q ≥ 0 and $\forall\, \lambda_1$, λ_2, $\lambda_3 \leq 0$, solving these equations, optimum quantities are

$$q = \frac{B + (1-\alpha) P_2}{A} = \frac{2(U + (1-\alpha) P_1)}{u}$$

$$D^* = \left[\frac{C_{03} q^{(\nu-1)} \pm C_{03}^2 q^{2(\nu-1)} - \frac{4K^2 C_1 q \beta (\beta - 1)}{2 \times 100}}{2(1-\beta) K} \right]^{-1/\beta}$$

q = f (α) and D = f (q) where α* is a root of

$$K \beta D^{*(1-\beta)} + \frac{1}{2 \times 100} C_1 K q^* D^{*-\beta} (1+\beta) - C_0 - (1-\alpha^*) P_0 = 0$$

$$C^*(D^*, q^*) = C_{03} q^{*\nu-1} D^* + K D^{*1-\beta} + \frac{1}{2 \times 100} C_1 K D^{*-\beta} q^*$$

So, by both FNLP and NLP techniques, the optimal values of q^* and D^* and the corresponding minimum cost are evaluated for the known values of other parameters.

V. NUMERICAL EXAMPLE

For a particular EOQ problem, let C_{03} = \$4, K = 100, C_1 = \$2, ν = 0.5, β = 1.5, u = \$0.5, U = \$3.5, A = 5 units, B = 90 units, C_0 = \$40 and $P_0 = \$20$ and P_1 =\$15 and $P_2 = 25$ units. For these values the optimal value of productions batch quantity q*, optimal demand rate D*, minimum average total cost C* (D*, q*) and Aq* obtained by FNLP are given in Table 2.

After 26 iterations Table-2 reveals the optimal replenishment policy for single item with demand dependent unit cost and dynamic setup cost. In this table the optimal numerical results of fuzzy model are compared with the results of crisp model and fuzzy model of Roy et al. (1997). The optimum replenishment quantity $q^*$and Aq^* are both 229.8354% and 82.45885% more than that of other crisp model and fuzzy model respectively, the optimum quantity demand D^* is 14.03017 but 9.21 and 9.81 for comparing models, hence 52.34% and 42.9972% more from the other crisp model and from the fuzzy model respectively. The minimum total average cost $C^*(D^*, q^*)$ is 40.83059 but 54.43 and 53.93 comparing models, hence -24.98513% and -24.2930% less from other crisp and fuzzy models respectively. It permits the better use of present fuzzy model as compared to the crisp model and other fuzzy model. The results are justified and agree with the present model. It indicates the consistency of the fuzzy space of EOQ model from other models.

Fig. 1 represents the relationship between demand per unit time D and unit cost of production P. Similarly Fig. 2 shows the relationship between number of order quantity q and variable setup cost C_3 and Fig. 3 depicts the mesh plot of demand per unit time D, number of order quantity q and average total cost C.

TABLE 2. OPTIMAL VALUES FOR THE PROPOSED INVENTORY MODEL

Model	Method	Iteration	q^*	D^*	$C^*(D^*, q^*)$	α^*	$\frac{1}{2}uq^*$	Aq^*
Fuzzy model	FNLP	26	16.49177	14.03017	40.83059	0.9584705	4.1229425	82.45885
Crisp model, Roy et al. (1997)	NLP	-	5	9.21	54.43	1	-	50
% Change	-	-	229.8354	52.3363	-24.98513	-4.15295	-	64.9177
Fuzzy model, Roy et al. (1997)	FNLP	-	6.0449	9.8115	53.9324	0.3033	-	60.449
% Change	-	-	172.82122	42.9972	-24.2930	216.0140	-	36.4106

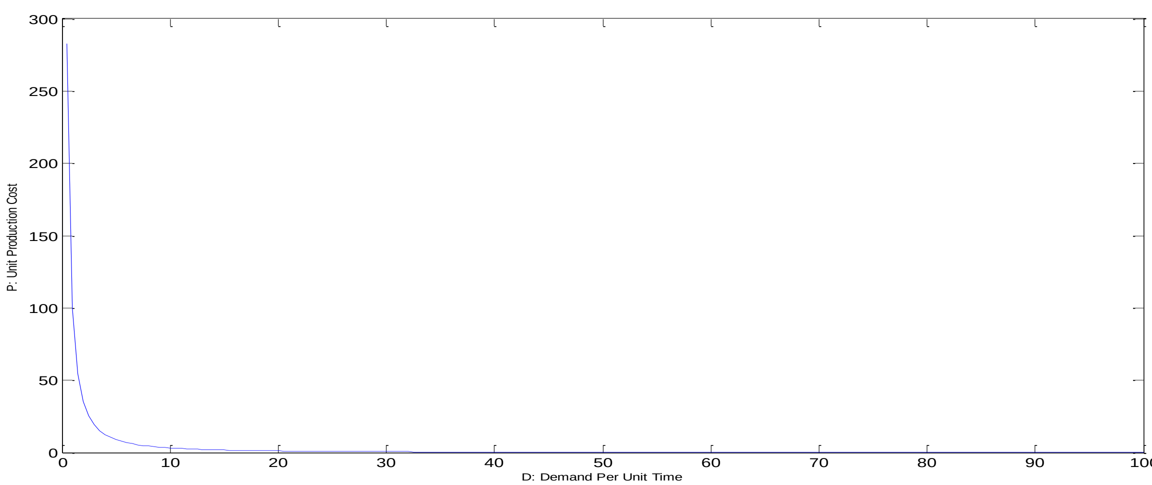

FIGURE 1. DEMAND PER UNIT TIME D AND UNIT PRODUCTION COST P

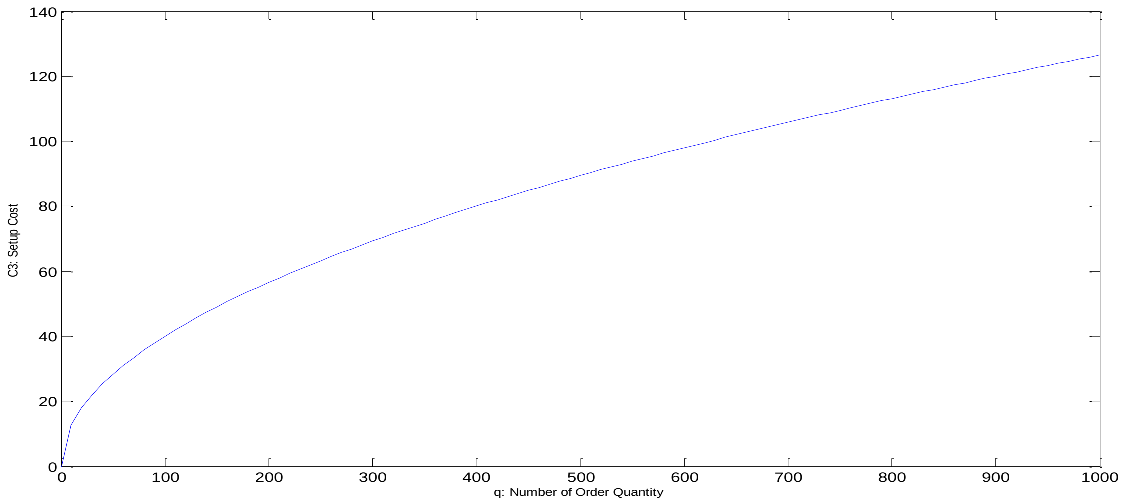

FIGURE 2. NUMBER OF ORDER QUANTITY q AND DYNAMIC SETUP COST C_3

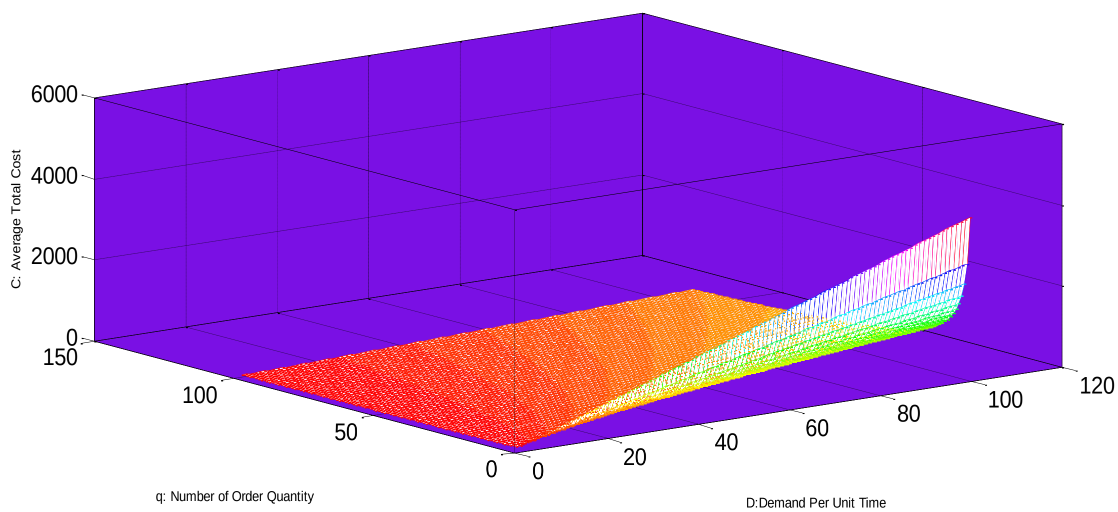

FIGURE 3. MESH PLOT OF DEMAND PER UNIT TIME D, NUMBER OF ORDER QUANTITY q AND AVERAGE TOTAL COST C

VI. SENSITIVITY ANALYSIS

Now the effect of changes in the system parameters on the optimal values of q, D, C (D, q) and $\frac{1}{2}uq$ and Aq when only one parameter changes and others remain unchanged the computational results are described in Table 3. As a result

$$\alpha^*,\ q^*, D^*,\ C^*(D^*, q^*), \frac{1}{2}uq^*$$

and A$q^*$are less sensitive to the parameters P_0,$P_1$and P_2. Following Dutta et al. [1] and Hamacher et al. [4] it is observed that the effect of tolerance in the said EOQ model with the earlier numerical values and construct Table 3 for the degrees of violation $T_0(=(1-\alpha)P_0), T_1$(=(1-α)P_1) and T_2 (=(1-α)P_2)for two constraints given by equation (6).

From Table 3, it is seen that: (i) For higher tolerances of P_0, the value of α_{max} does not achieve 1, (ii) For higher acceptable variations P_0, the optimal solutions remain invariant and the optimal solutions are very close to the solutions (q^* =16.49177, $D^* = 14.03017, C^*(D^*, q^*) = 40.83059, \frac{1}{2}uq^* = 4.1229425$ $and\ Aq^* = 82.45885$) of fuzzy model and (q^* =5, $D^* = 9.21, C^*(D^*, q^*) = 54.43$ $and\ Aq^* = 50$) of the crisp model without tolerance ($\alpha = 1$) respectively.

From Table 3 it is shown that: (i) For different values of P, degrees of violations T_0 , $T_1 and\ T_2$ are never zero, i.e. different optimal solutions are obtained. (ii) As $P_0, P_1$and P_2 increase from original values, the minimum average cost $C^*(D^*, q^*)$increases, decreases and remaining constant respectively, but q^* and D^*increase, decrease and remain stable respectively.

TABLE 3. SENSITIVITY ANALYSIS OF THE PARAMETERS P_0, $P_1$and P_2

P	Value	Iteration	α^*	q^*	D^*	T_0	T_1	T_2	$C^*(D^*, q^*)$	$\frac{1}{2}uq^*$	Aq^*
	25	24	0.965	16.239	13.955	0.9328	0.5597	0.9328	40.93276	4.0598	81.193
	50	27	0.975	15.494	13.729	1.2447	0.3734	0.6224	41.24472	3.8735	77.468
P_0	100	30	0.985	14.903	13.545	1.5049	0.2257	0.3762	41.50494	3.7258	74.515
	200	26	0.992	14.506	13.418	1.6868	0.1265	0.2108	41.68675	3.6265	72.530
	1000	29	0.998	14.112	13.291	1.8729	0.0281	0.0468	41.87295	3.5280	70.562
	16	34	0.9599	16.565	14.051	0.8015	0.6412	1.0019	40.80148	4.1413	82.8237
	20	36	0.9648	16.814	14.125	0.7034	0.7034	0.8792	40.70338	4.2035	84.0675
P_1	23	38	0.9678	16.965	14.169	0.6445	0.7412	0.8057	40.64453	4.2413	84.8242
	38	46	0.9772	17.462	14.312	0.4555	0.8655	0.5694	40.45552	4.3655	87.3096
	40	45	0.9781	17.508	14.325	0.4385	0.8769	0.5481	40.43845	4.3770	87.5382
	30	35	0.9585	16.492	14.030	0.8306	0.6230	1.2459	40.83059	4.1230	82.4589
	40	29	0.9585	16.492	14.030	0.8306	0.6230	1.6612	40.83059	4.1230	82.4589
P_2	50	29	0.9585	16.492	14.030	0.8306	0.6230	2.0765	40.83059	4.1230	82.4589
	80	24	0.9585	16.492	14.030	0.8306	0.6230	3.3224	40.83059	4.1230	82.4589
	100	26	0.9585	16.492	14.030	0.8306	0.6230	4.1530	40.83059	4.1230	82.4589

TABLE 4. SENSITIVITY ANALYSIS OF THE PARAMETERS u, U, A, B, C_1, C_{03} AND K

Parameter	Value	Iteration	D^*	q^*	$C^*(D^*,q^*)$	% Change in $C^*(D^*,q^*)$	$\frac{1}{2}uq^*$	Aq^*
u	1	27	12.46570	11.72621	43.150851	-5.3770921	5.863105	58.63105
	2	27	10.91410	7.967578	45.95677	-11.154352	7.967578	39.83789
	3	26	10.04213	6.245959	47.82525	-14.625454	9.368939	31.2298
	5	25	8.999078	4.526525	50.42175	-19.021871	11.316313	22.63263
	10	28	7.700481	2.858345	54.38897	-24.928547	14.291725	14.29173
U	4	31	14.29954	17.41700	40.47233	0.88519737	4.35425	87.085
	5	29	14.48128	18.05917	40.23666	1.47609170	4.5147925	90.29585
	20	29	14.48128	18.05917	40.23666	1.47609170	4.5147925	90.29585
	30	29	14.48128	18.05917	40.23666	1.47609170	4.5147925	90.29585
	50	29	14.48128	18.05917	40.23666	1.47609170	4.5147925	90.29585
A	10	23	11.62476	9.574816	44.59853	-8.4485744	2.90619	47.87408
	15	25	10.24085	6.614806	47.37767	-13.818915	2.5602125	33.07403
	30	24	8.265069	3.522763	52.54631	-22.295899	2.06626725	17.61382
	50	27	7.068062	2.217903	56.71611	-28.0088321	1.7670155	11.08952
	60	30	6.685926	1.880896	58.28301	-29.9442668	1.6714815	9.40448
B	150	29	14.03017	16.49177	40.83059	0	3.5075425	82.45885
	200	27	14.03017	16.49177	40.83059	0	3.5075425	82.45885
	250	26	14.03017	16.49177	40.83059	0	3.5075425	82.45885
	400	26	14.03017	16.49177	40.83059	0	3.5075425	82.45885
	1000	26	14.03017	16.49177	40.83059	0	3.5075425	82.45885
C_1	3	26	14.24903	16.70619	40.74676	0.2057341492	3.5622575	83.53095
	4	32	14.46739	16.92149	40.66632	0.4039460664	3.6168475	84.60745
	5	27	14.68532	17.13767	40.5891	0.594962687	3.67133	85.68835
	7	25	15.12015	17.57268	40.44368	0.956663686	3.7800375	87.8634
	10	28	15.77063	18.23174	40.24615	1.452163747	3.9426575	91.1587
C_{03}	5	22	12.70034	18.78197	40.19735	1.575327727	3.175085	93.90985
	6	32	11.41916	19.41162	40.46288	0.908758843	2.85479	97.0581
	7	26	10.44439	19.97335	40.88243	-0.126802638	2.6110975	99.86675
	9	27	9.043628	20.95129	41.9263	-2.613419262	2.260907	104.7565
	10	34	8.518795	21.38577	42.49042	-3.906362893	2.1296875	106.9289
K	110	25	20.86033	18.65744	41.40823	-1.394988388	5.2150825	93.2872
	120	24	16.66990	19.23358	39.97929	2.129352472	4.167475	96.1679
	140	29	18.79223	20.32908	39.98928	2.103838829	4.6980575	101.6454
	150	26	19.83246	20.85224	40.06344	1.914838067	4.958115	104.2612
	200	27	15.58482	18.65744	40.06637	1.907385171	3.896205	93.2872

Now the effect of changes in the system parameters on the optimal values of q, D, C (D, q), $\frac{1}{2}uq$ and Aq when only one parameter changes and others remain unchanged the computational results are described in Table 4. As a result

- q^*,D^*, $C^*(D^*,q^*)$, $\frac{1}{2}uq^*$ and Aq^* are highly sensitive to the parameter 'u'.
- $q^*, D^*, C^*(D^*,q^*)$, Aq* and $\frac{1}{2}uq^*$ are moderately sensitive to the parameter 'U'.
- $q^*, D^*, C^*(D^*,q^*), Aq^*$ and $\frac{1}{2}uq^*$ are highly sensitive to the parameter 'A'.
- $q^*, D^*, C^*(D^*,q^*), Aq^*$ and $\frac{1}{2}uq^*$ are insensitive to the parameter 'B'.
- $q^*, D^*, C^*(D^*,q^*), Aq^*$ and $\frac{1}{2}uq^*$ are insensitive to the parameter 'C_1' .

- $q^*, D^*, \frac{1}{2}uq^*$ and Aq^* are sensitive to the parameter 'C_{03}' but $C^*(D^*, q^*)$ are moderately sensitive to 'C_{03}'.
- q^*, $\frac{1}{2}uq^*$ and Aq^* are sensitive to the parameter 'K' but D^* and $C^*(D^*, q^*)$ are moderately sensitive to 'K'.

VII. CONCLUSION

In constraint to Roy, the approach in this paper provides solutions better than those obtained by using properties and this paper follows real life inventory model for single item in fuzzy environment by FNLP technique. Some sensitivity analyses on the tolerance limits have been presented. The results of the fuzzy model is compared with that of crisp model which reveals that fuzzy model gives better result than the usual crisp model. Inventory modelers have so far considered auto are type of setup cost that is fixed or constant. This is rarely seen to occur in the real market. In the opinion of the author, an alternative (and perhaps more realistic) approach is to consider the setup cost as a function quantity produced / purchased may represent the tractable decision making procedure in fuzzy environment. A new mathematical model is developed and numerical example is provided to illustrate the solution procedure. The new modified EOQ model was numerically compared to the traditional EOQ model. Finally, the effect decision space was demonstrated numerically to have an adverse affect on the total average cost per unit. This method is quite general and can be extended to other similar inventory models including the ones with shortages and deteriorate items.

REFERENCES

D. Dutta, J.R. Rao, R.N. Tiwari. Effect of tolerance in fuzzy linear fractional programming. *Fuzzy Sets and Systems*, 1993, 55: 133 - 142.

G. Sommer. Fuzzy inventory scheduling. In: G. Lasker (ed.). *Applied Systems and Cybernatics*, VI, Academic press, New York, 1981.

G. Urgeletti Tinareli. Inventory Control Models and Problems. *European Journal of Operational Research*, 1983, 14: 1 - 12.

H. Hamacher, H. Leberling, H.J. Zimmermann. Sensitivity analysis in fuzzy linear programming. *Fuzzy Sets and Systems*, 1978, 1: 269 - 281.

H.A. Taha. Operations Research - An introduction. 2nd edn. *Macmilliion*, New York, 1976.

H.J. Zimmermann. Description and optimization of fuzzy systems. *International Journal of General System*, 1976, 2: 209 - 215.

H.M. Lee, J.S. Yao. Economic Production quantity for fuzzy demand quantity and fuzzy production quantity. *European Journal of Operational Research*, 1998, 109: 203 - 211.

H.W. Kuhn, A.W. Tucker. Nonlinear Programming. In J. Neyman (ed.). *Proceedings Second Berkely Symposium and Mathematical Statistics and Probability*. University of California Press, 1951, 481 - 494.

J. Kacprazyk, P. Staniewski. Long term inventory policy - making through fuzzy decision making models. *Fuzzy Sets and Systems*, 1982, 8: 17 - 132.

K.S. Park. Fuzzy set theoretic interpretation of economic order quantity. *IEEE Transactions on Systems, Man, and Cybernetics SMC-17/6*, 1987, 1082 - 1084.

L.A. Zadeh. Fuzzy Sets. *Information and Control*, 1965, 8: 338 - 353.

M. Pattnaik. A Note on Non Linear Profit-Maximization Entropic Order Quantity (EnOQ) Model for Deteriorating Items with Stock Dependent Demand Rate., *Operations and Supply Chain Management*, 2012, 5 (2): 97-102.

M. Pattnaik. Effect of deteriorating items and promotional effort factor in fuzzy instantaneous replenishment model. *International Journal of Management Science and Engineering Management*, 2012, 7 (4): 303-311.

M. Vujosevic, D. Petrovic, R. Petrovic. EOQ Formula when inventory cost is fuzzy. *International Journal Production Economics,* 1996, 45: 499 - 504.

P.K. Tripathy, M. Pattnaik, P. Tripathy. Optimal EOQ Model for Deteriorating Items with Promotional Effort Cost. *American Journal of Operations Research*, 2012, 2 (2): 260-265.

P.K. Tripathy, M. Pattnaik. A non-random optimization approach to a disposal mechanism under flexibility and reliability criteria. *The Open Operational Research Journal,* 2011, 5: 1-18.

P.K. Tripathy, M. Pattnaik. An entropic order quantity model with fuzzy holding cost and fuzzy disposal cost for perishable items under two component demand and discounted selling price. *Pakistan Journal of Statistics and Operations Research*, 2008, 4 (2): 93-110.

P.K. Tripathy, M. Pattnaik. Optimal disposal mechanism with fuzzy system cost under flexibility and reliability criteria in non-random optimization environment. *Applied Mathematical Sciences,* 2009,3 (37): 1823-1847.

P.K. Tripathy, P. Tripathy, M. Pattnaik. A fuzzy EOQ model with reliability and demand dependent unit cost. *International Journal of Contemporary Mathematical Sciences*, 2011, 6 (30): 1467-1482.

R.E. Bellman, L.A. Zadeh. Decision making in a fuzzy environment. *Management Science*, 1970, 17: B141 - B164.

T.C.E. Cheng. An economic order quantity model with demand - dependent unit cost. *European Journal of Operational Research*, 1989, 40: 252 - 256.

T.K. Roy, M. Maiti. AFuzzy EOQ model with demand dependent unit cost under limited storage capacity. *European Journal of Operational Research*, 1997, 99: 425 - 432.

T.K. Roy, M. Maiti. A fuzzy inventory model with constraint. *Operational Research Society of India*, 1995, 32 (4): 287 - 298.