Optimal and Heuristic Production Planning in Battery Manufacturing

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Battery production is a multi-product, multi-component, and multi-stage manufacturing process. The trade-offs involved include material ordering and inventory holding costs, production setup cost, in-process and final product inventory holding costs, and cost of defects. We present a mixed integer programming (MIP) model to plan production at minimum cost for a selected planning horizon. Even though the model yields the optimal solution, the computing times are excessive. Therefore, we present an efficient heuristic based on a material requirements planning (MRP) structure to provide feasible solutions. We apply the MIP and the heuristic to plan production for a lead-acid battery plant, and compare the results.

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I. INTRODUCTION

This paper deals with production planning of products manufactured and assembled in multiple stages over an extended planning horizon. The manufacturing operation involves batch, continuous, and discrete item processes. The complexity of this environment increases with the need to switch among different processing modes, while satisfying production capacities and customer demand. Moreover, the quality of the outputs from certain manufacturing stages is impacted by the production planning decisions, such as run size. The situation is further complicated by the need to handle three types of inventories: in process, raw material, and finished products. The plant needs to explore the trade-offs among alternative lot-sizing costs such as ordering, setup, and inventory holding costs.

We develop and apply a mixed integer programming (MIP) model for the minimum cost production plan in a medium size lead-acid starter battery manufacturing plant. The model integrates all issues dealing with production and storage capacities, demand, inventory, and quality in selected stages at this plant, reflecting the special characteristics of the battery manufacturing process.

The quest for optimality in planning production for such a complex situation is demanding, particularly for small-to-medium size battery plants with limited availability of expertise or computational resources, as was the case in the plant studied by the authors. In addition to the need for skilled analysts to run mathematical programming software, the MIP model requires several hours of computational time on a PC. We therefore introduce a heuristic that combines lot sizing with material requirements planning (MRP) concepts for purchased and assembled parts. The heuristic uses forward loading to provide a feasible production plan. The quantity of each battery type produced during regular time and overtime, as well as the setup, inventory, and overtime costs, are compared between production plans developed by the two approaches. The sensitivities of both approaches to demand fluctuations and capacity utilization are also investigated.

The battery characteristics and its manufacturing process are described in the next section. In Section 3, we define the production planning problem in the lead-acid battery manufacturing plant. In section 4, we review the relevant literature. The MIP formulation is presented in section 5. Section 6 develops the heuristic procedure. Computational results are given in section 7. The paper ends with concluding remarks in section 8.

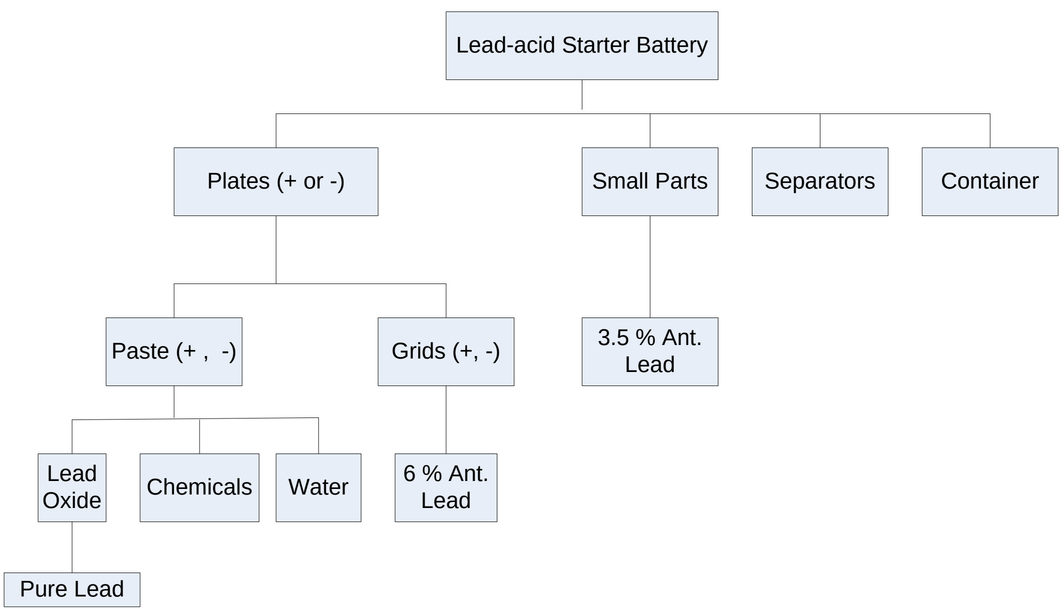
II. PRODUCT AND MANUFACTURING PROCESS DESCRIPTION

As shown in the bill of materials (Figure 1), a lead-acid battery (LAB) consists of several cells (containing positive and negative plates with insulating separators in between), a container and its cover, and small parts (connectors and posts). A cell is a small direct current source. An assembled battery is filled with sulfuric acid, covered with vent plugs and charged before its installation in a vehicle. Figure 1 shows that the containers, separators, positive and negative chemicals, pure lead, 3.5% antimony lead and 6% antimony lead represent the materials purchased from outside suppliers. The LAB manufacturing process (Figure 2) consists of: lead oxidation, small parts (connectors and posts) and grid casting, paste mixing and pasting, curing, formation, washing and drying, and assembly. The manufacturing stages are described below:

Material Supplies. The plant acquires pure lead, 6% antimony lead, and 3.5% antimony lead, containers, covers, vent plugs, separators, chemicals and sulfuric acid from outside suppliers.

Oxidation Stage. Pure lead is converted into lead oxide, packed into 180-kg barrels. A barrel is accepted or rejected based on the average percentage of pure lead and the oxide powder homogeneity. This process requires preheating and adjustment before reaching steady-state. The quality of the first and last oxide barrels deviates from specifications and these barrels are usually rejected (Elimam and Sartawai, 1986). This is a continuous process and its lead oxide output is storable.

FIGURE 1: BILL OF MATERIALS.



Small Parts Casting Stage. Connectors, posts, and terminals are cast from 3.5% antimony lead. The small parts casting machines are shut down for mold changeover and process parameter adjustment. This is a batch process and its output is storable.

Grid Casting Stage. 6% antimony lead is cast into negative or positive plate grids with the specified weight and thickness. A setup time is required for mold changeover depending on grid or battery type. This is a batch process and its output is stored.

Paste Mixing and Pasting. Lead oxide powder is mixed with sulfuric acid, water, and positive or negative chemicals. The paste mixer must be cleaned before switching between positive and negative paste. A change in plate type (+ or -) requires setup time. The pasted plates are air cured for 3 to 15 days depending on outdoor climate. This is a batch process and its output is cured while being stored.

Formation Lines. The cured plates are charged for 48 hours in formation lines. The plates' formation time, once loaded, is independent of the batch size. Equally charged cells and absence of sulfated plates are essential for a successful formation output. This is a batch process.

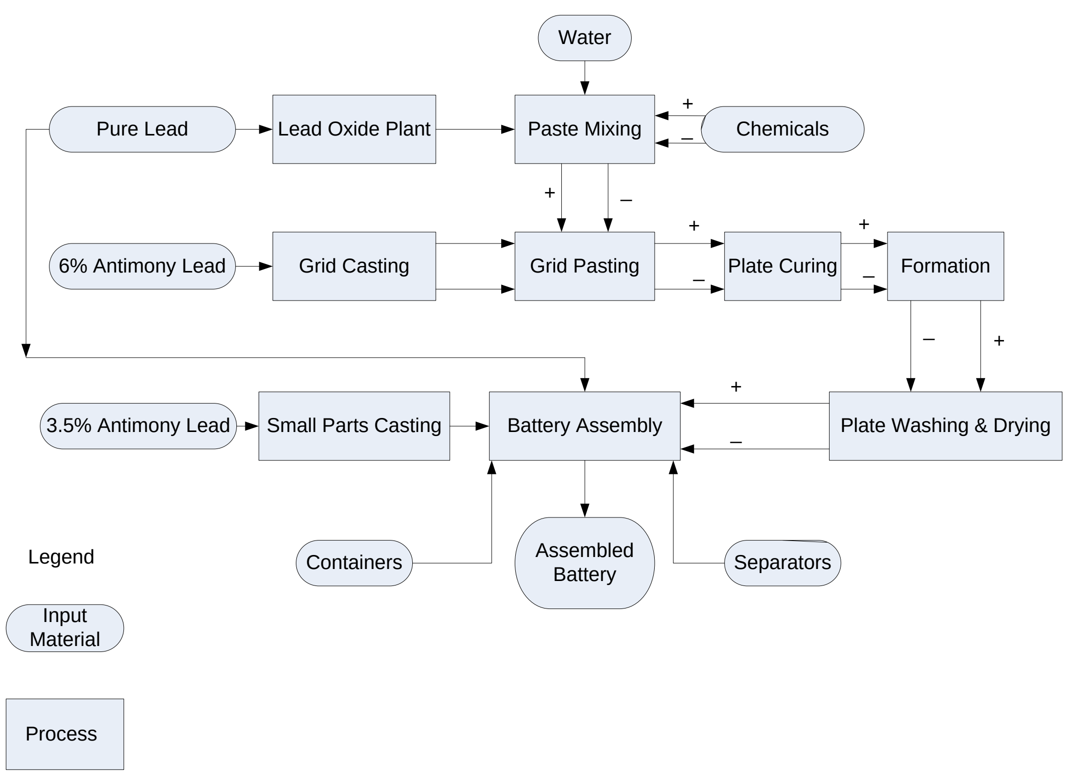
Washing. Charged plates are washed as soon as they are taken out of the formation lines. A setup time is required to load each batch. This is a batch process. Only positive plates are stored until assembly.

Negative Plate Dryer. Plates are loaded in batches into the dryer. This is a batch process and its output is storable.

Assembly. A battery cell is formed by stacking a sequence of negative plate, separator, and positive plate. All positive (negative) plates are welded together. At least two cells (depending on the battery type) are welded in series. Connections from the first and the last cells represent the negative and positive battery terminals. The stage setup and production time are dependent on battery type. This is a batch process and the assembled batteries are stored.

The process is shown in Figure 2 below.

FIGURE 2: MANUFACTURING PROCESS



III. PROBLEM DEFINITION

Even though production planning of assembled products has been frequently treated in the literature, lead-acid battery production planning has many unique features leading to challenging cost trade-offs.

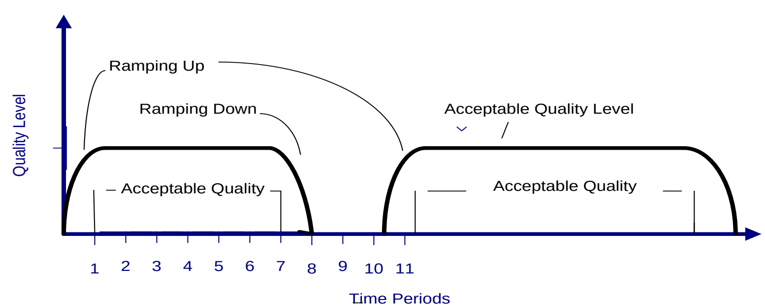
Inventory. All materials are supplied to the plant based on purchase orders. Since many of the purchased items are imported, sufficient stock must be maintained to avoid plant operation interruptions. Conversely, overstocking material can be expensive, tying up excessive capital and requiring larger storage capacity. Assembled batteries are either sold or stored as inventory to meet future demand. Due to the different battery types and components, the plant must resort to batch processing in several of the stages.

Manufacturing Process Type. The LAB manufacturing process includes batch operations such as the lead oxide, paste mixing, formation, and washing and drying stages. The first two stages provide continuous output while the last two produce discrete items. The grid and small parts casting and the assembly stages produce discrete items. The grid pasting stage combines the outputs of discrete and batch operations into a discrete item (plates). Such diversity in the manufacturing stages requires close synchronization among these stages.

Setup Time. The small parts and grid casting stages require setup time to switch molds for each battery type. The paste mixing stage requires setup time to clean and switch between positive and negative pastes. Jigs and fixtures also incur setup time to change molds for assembly.

Quality. In the oxidation stage, the lead oxide produced during the ramping up/down periods of a production run is of inferior quality (Figure 3). Defective lead oxide could be reduced by extending production runs and storing the produced lead oxide for future use. Unfortunately, this strategy would increase the inventory holding costs. Given the above environment, it is required to plan production of lead-acid batteries and its associated material requirements such that the total production cost is minimized while satisfying market demand over the planning horizon.

FIGURE 3: IMPACT OF PRODUCTION RUN LENGTH ON QUALITY



IV. LITERATURE REVIEW

The literature is rich with articles dealing with lot sizing as well as production planning and scheduling. In this section, we discuss several relevant contributions. It may be noted, as outlined in Section 1, that the battery manufacturing process has certain distinctive characteristics that distinguish it from other applications.

Dong et al. (2010) formulate a multi-objective mathematical programming model for the integrated charge planning problem in the context of a steel plant. They develop two new meta-heuristics, and perform computational experiments to show that their model is both feasible and effective compared to other standard approaches. Pastor et al. (2009) present an MIP (mixed-integer linear programming) model to develop a minimum cost production plan for a wood-turning company. The problem involves different product families/types with minimum respective lot sizes, specialized lathes, inter-family as well as intra-family setup times, and use of overtime as well as subcontracting. Absi and Kedad-Sidhoum (2008) formulate an MIP model for a multi-item capacitated lot-sizing problem with setup times and shortage costs. They propose some classes of valid inequalities, and employ combinatorial separation algorithms in a branch-and-cut framework to solve the problem. Lin (2007) develops a mathematical programming model for multiple integrated MRP production stages, and uses it to simultaneously obtain the master production schedule, the material requirements plan, the production schedule, and the capacity plan. Chen and Peng (2007) present an MIP model for integrating production planning with shop scheduling. Their model considers capacity constraints, operation sequences, lead times and due dates in a multi-order environment, and aims to minimize the cost of both production idle time and order tardiness penalty. Omar and Teo (2007) introduce a three-level hierarchical production planning and scheduling approach for multiple products with identical parallel machines in a batch processing environment. They use integer programming models at each level to successively break down aggregate plans into production sequences. Clark (2003) formulates the planning for a canning line at a beverage manufacturer as an MIP model. The author compares several heuristics using real data to illustrate the trade-offs between solution quality and computing time. The two best methods make hybrid use of local search and MIP. Alfieri et al. (2002) apply trivial LP-based rounding heuristics to the capacitated lot-sizing problem (CLSP). They indicate that unlike other approaches such as the Lagrangian relaxation, the LP-based heuristics can be easily extended to cope with complicating features, for example, when dealing with master production scheduling within an MRP system. They also use strong model formulations, like the Plant Location Formulation (PLF) or the Shortest Path Formulation (SPF), to obtain good results for a CLSP. Balakrishnan and Vanderbeck (1999) develop an optimization procedure to support tactical planning in a high mix, low volume electronics assembly plant. The model assigns product families to assembly lines to minimize setup cost on the placement machines while ensuring that the facilities are not overloaded. They formulate an integer program for the tactical planning problem and use a column generation approach to develop heuristics and lower bounds for the general problem. Katok et al. (1998) introduce a heuristic for finding good, feasible solutions for multi product lot sizing problems with general assembly structures, multiple constrained resources, and nonzero setup costs and setup times. Their algorithm exploits the special structure of the mathematical programming model, for the one-to-one link between a subset of the continuous variables and the set of binary variables. Gunasekaran et. al. (1998) develop a mathematical model for determining the optimal lot-sizes for a set of products and the capacity required to produce them in a multi-stage production system. They use such a model to support capacity planning at the production function level. Their model minimizes the total system cost per unit time, including set-up cost, cost due to the queuing of batches, and hiring cost of the machines. Huang and Xu (1998) discuss the modeling of aggregate scheduling problems in multi-stage and multi-item dynamic manufacturing systems with storage space limitations and production capacity constraints of workstations. They show that the model can be transformed into an equivalent static job assignment problem with multiple job classes over a space-time network. Finally, they propose a network algorithm to solve the equivalent problem.

V. PRODUCTION PLANNING MODEL

The MIP production planning model is developed below. The constraints include: regular and overtime production capacities, material and finished battery inventory limits, conservation of material flow, battery type changeover, material ordering and demand requirements. The objective function minimizes the total cost of production.

5.1. Notation

Table 1 lists the material types and the corresponding indices. The set M_1 contains materials independent of battery type materials. The set M_2 includes all materials dependent on battery type. M is the union of M_1 and M_2; it is also the total number of materials. Table 2 lists the production stage names, indices and output units. The set S_1 contains all stages independent of battery type. The set S_2 includes all stages dependent on battery and plate type. S is the union of S_1 and S_2; it is also the total number of manufacturing stages.

TABLE 1: TYPES, INDICES AND UNITS OF MATERIALS

Type of Material	Index m	Units	Set
Pure lead	1	kg	M_1
3.5 % antimony lead	2	kg	M_1
6.0 % antimony lead	3	kg	M_1
Positive plate chemicals	4	kg	M_1
Negative plate chemicals	5	kg	M_1
Separators	6	number	M_2
Containers	7	number	M_2

TABLE 2: NAMES, INDICES AND UNITS FOR PRODUCTION STAGES

Production Stage	Index s	Units of output	Set Definition
Small Parts	1	Kg	S_1
Lead Oxide	2	Kg	S_1
Paste Mixing	3	Kg	S_2
Grid Casting	4	No. of grids	S_2
Grid Pasting	5	No. of pasted grids	S_2
Plate Curing	6	No. of cured plates	S_2
Formation	7	No. of charged plates	S_2
Washing and Drying	8	No. of plates	S_2
Assembly	9	No. of batteries	S_1

a. Parameters

- Supplied Material and Parts

n	Number of battery types
a_{mj}	Material m amount needed for battery type *j* for $m \ \varepsilon$ M, number of units, Table 1.
b_j	Lead oxide amount needed for battery type j positive and negative plates, kg/battery.
d_{ej}	Number of positive (e=1) or negative (e=2) plates in battery type *j*, number/battery.
g_{ej}	Amount of positive (e=1) or negative (e=2) paste in battery type *j*, kg/battery.
f_j	Factor to convert battery type *j* to the standard battery (in terms of size and production time), ratio.
W	Defective lead oxide weight, kg/batch.

- Material Cost

CO_{mj}	Material m ordering cost for battery type j for $m \ \varepsilon$ M, \$/order.
CH_{mj}	Material m inventory holding cost for battery type *j* for $m \ \varepsilon$ M, \$/unit/period.

- Production Process Stages

CP_{sk}	Production stage *s* capacity during regular/overtime (*k=0/1)*, hours.
r_{sj}	Stage s production rate of battery type j components,for $s \ \varepsilon \ S$, number of units/hour,Table 2.
CI_s	Incremental overtime labor/machines cost in stage *s*, $s \ \varepsilon \ S$, \$/ hour.
CX	Lead oxide cost, \$/kg
CS_{esj}	Stage s setup cost for positive (*e=1*) or negative (*e=2*) plates for battery type *j*, for $s \ \varepsilon \ S_2$, \$/setup.
CE_{sj}	Stage s setup cost for battery type *j*, for $s \ \varepsilon \ S_1$, \$/setup.
CV_{sj}	Battery type j in-process inventory holding cost after stage *s*, $s \ \varepsilon \ S$, \$/unit/period.

- Storage Capacity

CW_s	Storage capacity after stage *s*, for $s \ \varepsilon$ S, number of units, Table 2
CR	Total material storage capacity

- Demand for Batteries

$D_j(t)$ Demand for battery type *j* during period *t*.

b. <u>Variables</u>

The material ordering and production setup switches are binary; the remaining variables are continuous.

- Supplied Material

$PM_{mj}(t)$ Material m amount for battery type j purchased during period *t*, for $m \varepsilon$ M, number of units, Table 1.

$IM_{mj}(t)$ Material m inventory level for battery type j at the end of period t for $m \varepsilon$ M, number of units, Table 1.

$\delta_{mj}(t)$ = 1 if material m for battery type *j* is ordered during period *t*, for $m \varepsilon$ M
= 0 otherwise

- Material Ordering Lot Sizes:

$QM_{mj}(t)$ Material m purchasing lot size for battery type *j* at the end of period *t* for $m \varepsilon$ M, number of units, Table 1.

- Production Stage Output

$P_{sj}(t)$ Amount produced in stage s for battery type *j* during period *t* for $s \varepsilon S_1$, number of units, Table 2.

$PP_{esj}(t)$ Stage *s* production of positive (*e=1*) or negative (*e=2*) plates for battery type *j* during period *t* for $s \varepsilon S_2$, number of units, Table 2.

$\beta_{sj}(t)$ = 1 if stage s is switched to produce battery type j or its components during period t for $s \varepsilon S_1$.
= 0 otherwise

$\alpha_{esj}(t)$ =1 if stage s is switched to produce positive (*e=1*) or negative (*e=2*) plates for battery type *j* during period *t* for $s \varepsilon S_2$.
=0 otherwise

- Production: In-Process Inventory

$I_{sj}(t)$ Inventory level of stage *s* output for battery type *j* at the end of period *t* for $s \varepsilon S_1$, number of units, Table 2.

$IP_{esj}(t)$ Inventory level of positive (*e=1*) or negative (*e=2*) plates for battery type *j* during period *t* for $s \varepsilon S_2$, number of units, Table 2.

- Production: Plant Overtime Hours

$OT_s(t)$ Stage *s* overtime working hours during period *t* for $s \ \varepsilon$ S.

5. 2. Model Statement

Using the preceding notation, the MIP model is formulated below.

A. Objective Function. The criterion of optimization includes the costs of setup, material procurement, inventory holding, overtime, and yield loss. The plant equipment and work force are available all year round. Therefore, production line and labor costs during regular shift operation are considered sunk costs. We only consider the incremental overtime cost as needed, the cost of lead oxide quality defects and the setup cost of production stages. The setup cost includes:

- cost of starting production in the lead oxide station, and
- cost of changeover in small parts casting, grid casting, and assembly stages.

Therefore our objective is to minimize the total cost, expressed as follows:

$$OTC = \sum_{t=1}^{T} TC(t)$$

where

TC (t) is given by the following expression:

$TC(t) =$ [Material ordering cost + holding cost] +[Overtime cost + Setup cost + Lead oxide defects]+ [Finished battery inventory cost + In-process inventory cost]

$$TC(t) = \sum_{j=1}^{n}\sum_{m\in M} CO_{mj}\cdot\delta_{mj}(t) + \sum_{j=1}^{n}\sum_{m\in M} CH_{mj}\cdot IM_{mj}(t) + \sum_{s=1}^{S} CL_s\cdot OT_s(t) + \sum_{j=1}^{n}\sum_{s\in S_1} CE_{sj}\cdot\beta_{sj}(t) + \sum_{e=1}^{2}\sum_{j=1}^{n}\sum_{s\in S_2} CS_{esj}\cdot\alpha_{esj}(t) + CX.\,W.\,\underset{j\in n}{Max}\{\beta_{2j}(t)\} + \sum_{j=1}^{n}\sum_{s\in S_1} CV_{sj}\cdot I_{sj}(t) + \sum_{e=1}^{2}\sum_{j=1}^{n}\sum_{s\in S_2} CV_{sj}\cdot IP_{esj}$$

B. Constraints: the following relationships apply for each time period t, t=1,…, T.

i. Material Inventory Balance: Material inventory level, in a given period, is updated by adding the amount of material purchased minus the amount consumed to make various types of batteries.

a. <u>Battery type-independent material</u>

$$\sum_{j=1}^{n} IM_{mj}(t) = \sum_{j=1}^{n} IM_{mj}(t-1) + \sum_{j=1}^{n} PM_{mj}(t) - \sum_{j=1}^{n} a_{mj}\cdot P_{sj}(t) \qquad \forall\ m\in M_1 \qquad (1)$$

b. <u>Battery type-dependent material</u>

$$IM_{mj}(t) = IM_{mj}(t-1) + PM_{mj}(t) - a_{mj}\cdot P_{sj}(t) \qquad \forall\ m\in M_2 \quad j=1,\cdots,n \qquad (2)$$

ii. Material Storage Capacity: The inventory level of all materials cannot exceed the material storage capacity.

$$\sum_{j=1}^{n}\sum_{m=1}^{M} f_j\cdot IM_{mj}(t) \le CR \qquad (3)$$

iii. Procurement of material: The amount of material purchased is bounded by the material ordering (0,1) switch multiplied by the storage capacity.

$$PM_{mj}(t) \le CR \cdot \delta_{mj}(t) \qquad \forall\ m \in M \qquad j = 1,\cdots,n \qquad (4)$$

iv. Production Capacities: The amount produced in stage s, based on the production rate, must be ≤ the total available capacity for this stage.

a. Plate independent stages

$$\sum_{j=1}^{n} \frac{P_{sj(t)}}{r_{sj}} \le \sum_{k=0}^{1} CP_{sk} \qquad \forall\ s \in S_1 \qquad (5)$$

b. Plate Manufacturing Stages

$$\sum_{j=1}^{n}\sum_{e=1}^{2}\left[\frac{PP_{esj}(t)}{r_{sj}}\right] \le \sum_{k=0}^{1} CP_{sk} \qquad \forall\ s \in S_2 \qquad (6)$$

v. Changeover of battery type and its parts: The production in stage s is the setup switch multiplied by the production rate and the total available capacity.

a. Plate independent stages

$$P_{sj}(t) \le \left(r_{sj} \cdot \sum_{k=0}^{1} CP_{sk}\right) \cdot \beta_{sj}(t) \qquad \forall\ s \in S_1 \qquad j = 1,\cdots,n \quad (7)$$

b. Plate Manufacturing Stages

$$PP_{esj}(t) \le \left(r_{sj} \cdot \sum_{k=0}^{1} CP_{sk}\right) \cdot \alpha_{esj}(t) \qquad (8)$$
$$\forall\ s \in S_2,\ e = 1,2 \qquad j = 1,\cdots,n$$

vi. Production Stages Overtime: Amounts produced in excess of regular production capacity is made during overtime hours.

$$OT_s(t) \ge \sum_{j=1}^{n} \frac{P_{sj}(t)}{r_{sj}} - CP_{s0} \qquad \forall\ s \in S_1 \quad (9)$$

$$OT_s(t) \ge \sum_{j=1}^{n}\sum_{e=1}^{2}\left[\frac{PP_{esj}(t)}{r_{sj}}\right] - CP_{s0} \qquad \forall\ s \in S_2 \quad (10)$$

vii. In-Process Inventory : In-process inventory is updated by adding the amount of production in stage s minus the amount consumed in the next stage. Negative and positive plates in-process inventories are monitored separately. In-process inventory after paste mixing in stage 3 equals zero. Mixed paste must be used immediately in the grid pasting of stage 5.

a. Battery Assembly Stage

$$I_{sj}(t) = I_{sj}(t-1) + P_{sj}(t) - D_j(t) \qquad j = 1,\cdots,n \qquad (11)$$

b. Lead Oxide Stage

$$\sum_{j=1}^{n} I_{3j}(t) = \sum_{j=1}^{n} I_{3j}(t-1) + \sum_{j=1}^{n} P_{3j}(t) - \sum_{j=1}^{n} b_j \cdot P_{3j}(t) \qquad (12)$$

c. Paste Inventory

$$\sum_{j=1}^{n} PP_{e4j}(t) - \sum_{j=1}^{n} g_{ej} \cdot PP_{5j}(t) = 0 \qquad e=1,2$$
$$IP_{e4j}(t) = 0 \qquad e=1,2 \qquad (13)$$

d. Pasted Plates In-Process Inventory

$$IP_{esj}(t) = IP_{esj}(t-1) + PP_{esj}(t) - PP_{e(s+1)j}(t) \quad e=1,2\ ,\ s = 4,\cdots,S-2 \qquad (14)$$
$$j = 1,\cdots,n$$

e. Charged Plates In-Process Inventory

$$IP_{e(s-1)j}(t) = IP_{e(s-1)j}(t-1) + PP_{e(s-1)j}(t) - \sum_{j=1}^{n} d_{ej} \cdot PP_{sj} \quad e=1,2 \ , \ j = 1,\cdots,n \tag{15}$$

viii. Non-negativity Constraints: All variables are nonnegative

The above model consists of [nT (3 M + 2|S_1|+ 4|S_2|) + ST] continuous variables (where |Z|is the cardinality of the set Z), [nT(M+|S_1|+ 2|S_2|)] binary variables, and T[6+ 3n+ 2n (S-6)+ |M_1|+ n|M_2|+ |M|+ (2+n)|S_1|+ (2+2n) |S_2|] constraints, in addition to the non-negativity constraints. This model was applied by the authors to plan the production of Lead-acid batteries in a medium size plant, using CPLEX optimization software.

VI. A HEURISTIC PROCEDURE

The MIP model described above captures the full complexity of the production process to develop an optimal production plan. However, the computing time is excessive even for relatively small problems. Further, it is inappropriate for a plant that does not have the services of an analyst to prepare the input files for the optimization software. Therefore, we develop a heuristic procedure to determine the production plan. The main features of the heuristic are described below.

- Lot sizes are determined for the final (assembly) stage using a variant of the Economic Production Quantity model. This is operationalized for the discrete time problem by using the POQ (Period Order Quantity) concept.
- Lot sizes at the preceding stages (including purchased materials) are then computed based on a Bill of Materials explosion.
- The lot sizes are modified to account for quality losses in the lead oxide stage.
- The lot sizing formula at the assembly stage includes a feedback loop from the preceding stages (including purchased materials) that serves to minimize the total lot sizing related costs over all stages.
- If the regular time production capacity is exceeded in any time period, then the cost impact of reducing the lot size for any of the batteries is evaluated by trading off the reduction in the overtime premium and the inventory carrying cost with the increase in the setup cost. Note that changing the lot size at any stage affects the lot sizes at the other stages as well.

6.1. Lot Sizing

To determine the lot sizes, we use a modified version of the Economic Production Quantity model. We first present the analysis in general terms, and then apply it to the battery context. We define the following variables:

D: mean demand rate/period
P: production rate/period
S: setup cost
C: unit inventory carrying cost/period
Q: lot size

The standard EPQ model makes the assumption that a new batch is started just when the inventory level reaches zero. This results in

Max Inventory = Q(1 - D/P) and
Min Inventory = 0,

So, Average Inventory = $\frac{Q}{2}(1 - D/P)$

We feel the above assumption is unrealistic. At the other extreme, we may assume that production of a new batch is *completed* just when the previous batch has been wholly consumed. In this case, since P>D, the minimum inventory occurs at the point where production of the new batch commences, so that Minimum inventory = DQ/P. Also, the maximum inventory occurs at the point where production of the new batch is completed, so Maximum inventory = Q.

Thus Average inventory = $\frac{Q}{2}(1 + D/P)$

Notice that in both cases, the difference between the maximum and minimum inventories is Q(1 - D/P).

We can model a whole range of situations in between these two extremes by incorporating a variable z ($0 \le z \le 1$). The standard EPQ model is obtained when z = 0, while the second model corresponds to a value of z = 1. This leads to

Average inventory = $\frac{Q}{2}\{1 + \frac{D}{P}(2z-1)\}$

Then the optimal lot size is given by

$$Q = \sqrt{\frac{2DS}{C\{1 + (2z-1)D/P\}}} \qquad (16)$$

Next we consider the effect of quality losses at the lead oxide stage. Typically, the first and last barrels of lead oxide in each batch are rejected due to poor quality. The quantity of lead oxide rejected is independent of the batch size. Thus the cost of the rejected lead oxide plays the same role as the setup cost. If W is the weight of lead oxide rejected per batch and CX is its unit cost, then the optimal lot size is given by

$$Q = \sqrt{\frac{2D(S + W.CX)}{C\{1 + (2z-1)D/P\}}} \qquad (17)$$

Finally, we modify the above model in the context of the bill of materials explosion. Suppose each unit of the end product uses n units of a component. Then the demand rate for the component is nD, while its lot size is nQ. We let S', C', and P' represent the setup/order cost, the unit inventory carrying cost, and the production rate (where appropriate) for the component. If the component is purchased, then the optimal lot size for the end product is given by

$$Q = \sqrt{\frac{2D(S + S' + W.CX)}{C\{1 + (2z-1)D/P\} + nC'}}$$

If the component is manufactured in-house, then the optimal lot-size for the end product is

$$Q = \sqrt{\frac{2D(S + S' + W.CX)}{C(1 + (2z-1)D/P) + nC'(1 + (2z-1)D/P')}}$$

Note that when the POQ equivalent is used in an MRP environment, the component is completely used up to produce the end product, resulting in an ending inventory of zero for the component. Then the above formulas reduce to

$$Q = \sqrt{\frac{2D(S + S' + W.CX)}{C(1 + (2z-1)D/P)}} \qquad (18)$$

We now apply the above models to the battery problem, using the notation listed in Section 5. Based on equation (18), we obtain the following lot size formula for batteries:

$$Q_j = \sqrt{\frac{2\bar{D}_j(\sum_{s \in S_1} CE_{sj} + \sum_{s \in S_2}\sum_{e=1}^{2} CS_{esj} + W.CX + \sum_{m \in M} CO_{mj}}{CV_{9j}\{1 + (2z-1)\bar{D}_j/(CP_{s0}.r_{9j})\}}} \qquad (19)$$

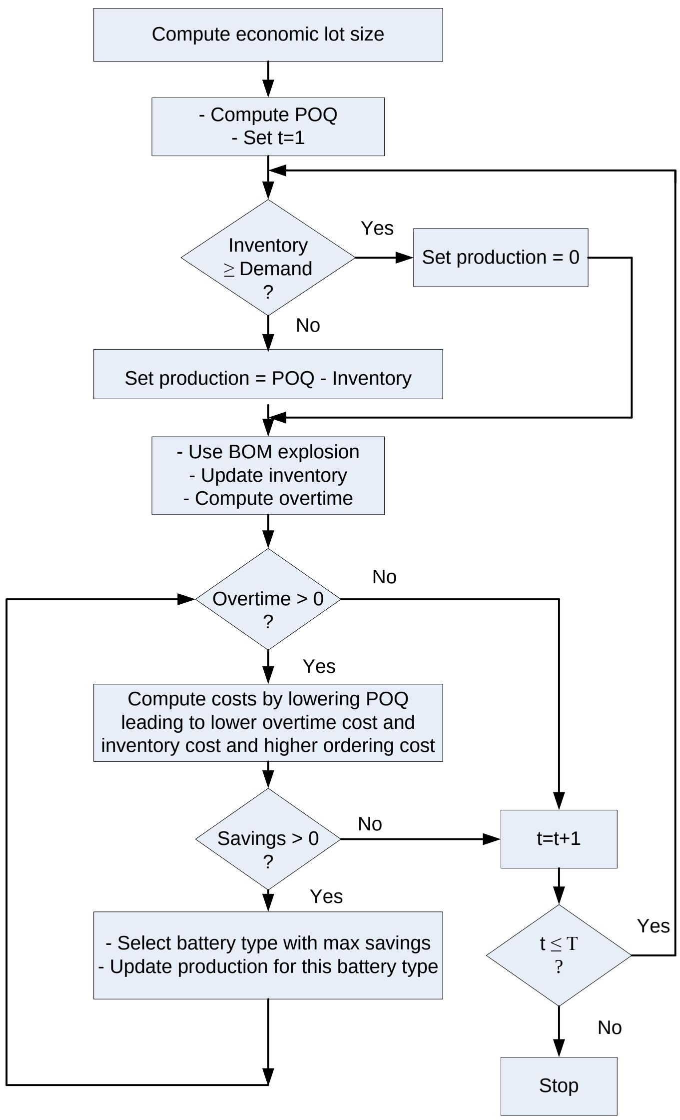
where $\bar{D}_j$ = mean demand for battery type j.

The lot sizes for purchased materials and for different manufacturing stages are determined based on the bill of materials explosion (Figure 1).

6.2 Production Planning

Next we consider the regular time production capacity at each stage. If this is exceeded in any time period, then we examine the effect of reducing the lot sizes for that period. Note that this will affect the lot sizes at all other stages as well. Reducing the lot size will result in a decrease in overtime costs and inventory carrying costs, while increasing the setup/order costs. We use the following heuristic for production planning (Figure 4).

FIGURE 4: PRODUCTION PLANNING HEURISTIC



1. Compute mean demands $\bar{D}_j$ for battery type j.
2. Compute lot sizes Q_j for battery type j using equation (19).
3. Compute the Period Order Quantity B_j for battery type j, where

$$B_j = \frac{Q_j}{\bar{D}_j}$$ rounded off to the nearest integer.

Repeat steps 4 - 10, for t = 1, ... , T.

4. Define set J = {j: j = 1,……,n}.
Determine the production of batteries, as follows:

If $I_{9j}(t-1) < D_{9j}(t)$, then

set $P_{9j}(t) = \sum_{k=t}^{t+B_j-1} D_{9j}(k) - I_{9j}(t-1)$;

else, set $P_{9j}(t) = 0$.

5. Based on a bill of materials explosion, compute the production at each stage, as well as material purchases. For example, for manufactured parts in stage $s \in S_1$, we have

$$P_{sj}(t) = \frac{D_{sj}(t)}{D_{9j}(t)} P_{9j}(t) ,$$

where $D_{sj}(t)$ is the demand in stage s for battery type j in period t.

6. Compute the ending inventory in period t. For example, for manufactured parts in stage $s \in S_1$, we have

$$I_{sj}(t) = I_{sj}(t-1) + P_{sj}(t) - D_{sj}(t)$$

7. Compute overtime hours required in stage s in period t. For example,

$$OT_s(t) = Max\{\sum_j \frac{P_{sj}(t)}{r_{sj}} - CP_{s0}, 0\} \quad \text{for} \quad s \in S_1$$

8. If $OT_s(t) = 0 \quad \forall$ s, then increment t by 1, and return to step 4. Else, go to step 9.
9. Evaluate the cost implications of reducing the overtime hours.

For each $j \in J$ with $B_j > 1$, compute the following:

- Decrease in overtime premium:

$$COT_j(t) = \sum_{s \in S} CI_s \frac{Min\{OT_s(t), D_{sj}(t+B_j-1)\}}{r_{sj}}$$

- Decrease in inventory carrying cost:

$$CIC_j(t) = CV_{9j}.D_{9j}(t+B_j-1)$$

- Increase in setup cost:

$$CSC_j(t) = \sum_{s \in S_1} CE_{sj} + \sum_{s \in S_2} \sum_{e=1}^{2} CS_{esj} + \sum_{m \in M} CO_{mj}$$

Then compute saving from reduction in overtime:

$$SV_j(t) = COT_j(t) + CIC_j(t) - CSC_j(t)$$

10. If $SV_j(t) \le 0 \quad \forall \ j \in J$, then increment t by 1 and return to step 4.

If $SV_j(t) > 0$, then it pays to reduce the batch size of product j in period t by the demand in period $(t + B_j - 1)$. Accordingly, reduce the batch size of product j (say j*) with highest $SV_j(t)$. For example, for manufactured parts in stage $s \in S_1$, we have

$$P_{sj*}(t) = P_{sj*}(t) - D_{sj*}(t + B_j - 1)$$

Delete j* from the set J, and return to step 7.

The above heuristic was programmed and run on a PC. Note that the heuristic is not computationally demanding.

VII. COMPUTATIONAL WORK

We describe the computational experiments and analyze the results below. The runs were based on actual data for the plant whose manufacturing process was described in this paper.

7.1 Computational Experiments:

In this section, we describe the input data and the computational runs used to develop production plans using both the MIP optimization and the heuristic procedure. In all our runs, we develop a 12-month production plan for three types of batteries. The procedures are compared based on six cost elements as well as the total cost, which is the metric used for evaluation.

First, we start with a base case (BC), where the plant is required to supply 6000, 1500 and 2400 units of battery types 1, 2 and 3 per year. In this case, the monthly distribution of this demand is based on real life data for the battery plant under study. To test the impact of monthly variation in demand on the amount produced and the cost of the plan, we make three more runs using different levels for the ratio of the minimum monthly demand over maximum monthly demand, namely: Min/Max =1, 0.6, and 0.2 (BCCD, BC6D, and BC2D respectively).

To test the impact of slack in plant capacity on the production plan, we also make 3 additional runs to achieve utilization levels of 100% (XUDC), 75% (75UDC) and 50% (50UDC) . In all of the seven computational experiments, we analyze the results of the heuristic and the MIP approaches, using the following elements of comparison:

- The above mentioned six elements of cost, as well as the total cost.
- The production plan (number of finished batteries produced by type every month).
- Finished battery inventory level, and
- Utilization of the regular and overtime plant capacity.

7.2. Comparison of the MIP and the Heuristic:

The MIP production plans exactly matched demand every month, while the heuristic employed lot sizing at lower levels of capacity utilization, Table 3. The extent of lot sizing was substantially reduced at high levels of utilization. As a result, the MIP had higher material order cost and production setup cost than the heuristic, Table 4. The difference in these costs became smaller at higher levels of capacity utilization. While both procedures had no material inventory holding costs, the heuristic had substantially higher work-in-process/finished goods holding cost. Again, the difference between the two procedures became smaller at high utilization levels. The heuristic made slightly higher use of overtime. The cost of defective lead oxide produced was of roughly the same magnitude in both procedures. Overall, the heuristic cost was about 8% higher on average than the MIP at low utilization levels (with a low of 5.5% and a high of 11%). This remained at 8% at 50% utilization, decreased to 5% at 75% utilization, and to 1% at 100% utilization. This may be explained by the fact that the problem is tightly constrained at high utilization levels, leaving the MIP with little flexibility in its search for the optimal solution.

In terms of computation time, the heuristic had a clear advantage, solving the problems in a matter of seconds versus several hours for the MIP.

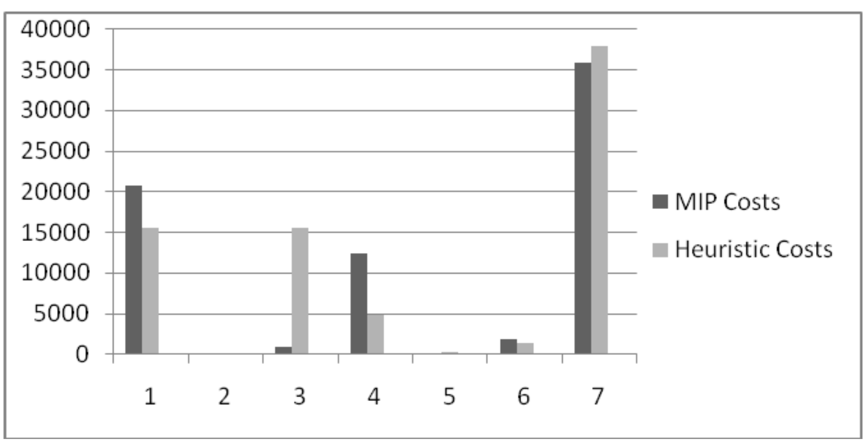
TABLE 3. MIP AND HEURISTIC MONTHLY PRODUCTION PLAN FOR BASE CASE DEMAND

Month	Demand per Type			MIP Plan			Heuristic Plan		
	I	II	III	I	II	III	I	II	III
Jan.	580	0	680	410	0	500	1070	0	950
Feb.	350	0	370	350	0	379	0	0	0
Mar.	310	140	80	310	70	80	0	360	0
Apr.	355	190	0	355	190	0	1085	0	0
May	250	100	110	250	100	110	0	0	630
Jun.	480	230	140	480	230	140	0	690	0
Jul.	840	250	180	840	250	180	2585	0	0
Aug.	945	210	200	945	210	200	0	0	0
Sep.	800	200	190	800	200	190	0	345	640
Oct.	650	100	180	650	100	180	1090	0	0
Nov.	245	45	140	245	45	140	0	0	0
Dec.	195	35	130	195	35	130	0	35	0

TABLE 4. PRODUCTION PLAN COSTS FOR SEVEN CASES USING MIP AND HEURISTIC

Case	Approach	Costs, $						
		Material ordering	Inventory holding		Production setup	Overtime	Defective lead oxide	Total
			Material	In process				
BC	MIP	20730	0	922	12420	0	1800	35872
	Heuristic	15610	0	15629	4870	375	1350	37833
BCCD	MIP	22060	0	302	13130	0	1650	37142
	Heuristic	15720	0	17021	5040	661	1200	39642
BC6D	MIP	22060	0	289	13130	0	1650	37129
	Heuristic	15720	0	17144	5040	1156	1200	40261
BC2D	MIP	21560	0	268	13280	0	1800	36908
	Heuristic	15720	0	17777	5040	1277	1200	41015
XUDC	MIP	22790	0	6768	12740	171731	1800	215829
	Heuristic	22390	0	8441	11080	174735	1800	218446
75UDC	MIP	22690	0	1625	12740	99608	1800	138463
	Heuristic	22280	0	8466	10610	102610	1650	145615
50UDC	MIP	22340	0	1143	12740	38016	1800	76039
	Heuristic	22170	0	7454	10290	40497	1650	82061

FIGURE 5: COMPARISON OF MIP AND HEURISTIC COSTS (BASE CASE)



A graphical comparison of the seven different cost elements of Table 4 is shown above for the base case (Figure 5). It may be noted that the major difference between the MIP and the heuristic is with respect to the in process inventory holding cost and the production setup cost. The same pattern occurs in all the runs, though the differences are less pronounced at high capacity utilization levels.

VIII. CONCLUDING REMARKS

In this paper, we have described two solution approaches to a practical problem with several cost tradeoffs, representing the ultimate in production planning complexity. The MIP formulation captures the essential problem elements and assumes a linear cost structure to give the optimal solution. The heuristic employs the EPQ model appropriately modified in several ways, and embeds it in an MRP framework. The two approaches were compared on a "real" base case, and sensitivity analysis was performed with respect to demand variability and capacity utilization levels. Overall, the heuristic performed reasonably well, especially at high capacity utilization levels. The MIP was quite demanding in terms of computational effort. The heuristic, however, was much simpler to use and required minimal computational effort. The heuristic is therefore useful for small to mid-size organizations with limited computational/ mathematical modeling resources, especially those operating at high capacity utilization levels.

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