The Impacts of Integrations and Demand Asymmetry on Industry Efficiency of Competing Supply Chains

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Consider an industry consisting of two supply chains. Each channel produces and distributes a competing product. Each channel can be either decentralized or integrated. Under this framework, we study the industry performance issue. Specifically, we explore how demand asymmetry, horizontal (differentiation) and vertical integrations impact industry performance. We found that (1) an industry with horizontal and vertical competition can yield a monopoly-like equilibrium, even though all parties involved pursue their own best interestswithout any collusion; and (2) for each industry configuration, there exists a unique degree of differentiation that maximizes industry profitability.

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I. INTRODUCTION

It is widely believed that locally optimal but decentralized decision making in supply chains leads to a suboptimal overall performance called double marginalization. This occurs when supply chain members aim to maximize their respective profits. By aligning the objectives of individual channel members, supply chain coordination or vertical integration can effectively eliminate double marginalization, and thus improve supply chain performance. Different coordination mechanisms have been explored, such as supply contracts, information technology, information sharing, and joint decision making (Arshinder et al., 2008).

Channel conflicts and coordination mechanisms have been studied extensively under a variety of supply chain structures. The study starts with the simplest dyadic supply chain with one upstream firm (typically called supplier or manufacturer) and one downstream firm (typically called buyer or retailer). Naturally, it has been extended to other more complex scenarios, such as multiple manufacturers with one or more common retailers, one manufacturer with multiple retailers or channels, and multiple manufacturers with multiple retailers or channels. Voluminous literature has been dedicated to this research. Summaries of this stream of literature can be found in (Cachon, 2003), (Cattani, Gilland and Swaminathan, 2004), (Tsay and Agrawal, 2004),(Li and Wang, 2007), (Arshinder, Kanda and Deshmukh, 2008), (Nagarajan and Sosic, 2008), and (Zhang, 2011). Typically in an industry, however, there exists multiple manufacturers, multiple products, and multiple distribution channels. In this scenario, how would coordination or integration of a single channel affect the performance of the entire industry? Our work considers the industry setting with two manufacturers, two competing products, and two exclusive channels, and it studies the possible impact of vertical integration of each channel on industry performance.

It probably is a good idea to integrate the distribution channel if firms are concerned with their respective supply chains alone. However it remains unclear how the integration of individual channel will affect the performance of the entire industry when multiple channels competing with each other. Our paper is to investigate the efficiency issue of an industry consisting of two competing channels.

We assume that each channel produces and distributes one competing product. Each channel can be either vertically decentralized or integrated. In a decentralized channel, there is one upstream firm (called manufacturer hereafter) and one downstream firm (called retailer hereafter). Each firm makes its decision independently to maximize its own profits. In an integrated channel, the two firms are merged together as one firm and this single firm makes all decisions by herself. According to different degrees of vertical integration across different channels, we thus have the following industry channel configurations: (1) decentralized configuration (denoted by DD) in which both channels are decentralized; (2) integrated configuration (denoted by II) in which both channels are integrated; and (3) mixed configurations (denoted by DI or ID) in which the first channel is decentralized and the second one is integrated or the vice versa. We also consider the case of fully centralized industry, i.e., one entity makes all decisions for the entire industry to maximize industry profit as the benchmark case (denoted by B).

Given the industry configurations, we calculate their equilibrium prices, quantities, and profits.We then study the issue of industry efficiency and investigate the impact of demand asymmetry, differentiation, and vertical integration. Industry efficiency is defined as the ratio of the equilibrium profit of any aforementioned industry configuration to the optimal profit of the benchmark case.

Key findings are: (1) an industry with horizontal and vertical competition can yield a monopoly-like equilibrium, even though all parties involved pursue their own best interestswithout any collusion; and (2) for each industry configuration, there exists a unique degree of differentiation that maximizes industry profitability.

The remainder of this paper is organized as follows. Related literature is reviewed in Section 2. Our model and equilibrium results are described in Section 3. Section 4 analyzes results from Section 3. Section 5 concludes the paper.

II. LITERATURE REVIEW

The industry configurations studied in this paper are similar to those in (McGuire and Staelin, 1983). They assumed static linear demand and cost functions. And they found that two manufacturers tend to integrate their distribution channels when two products are less substitutable. Moorthy (1988) later pointed out that demand dependence and strategic dependence are important for decentralization to occur as an equilibrium strategy. Wu and Mallik (2010) extended this line of research. They allowed retailers choose to carry competing products and identified the conditions for cross sales to occur. This stream of research focuses on strategic interactions between and within competing channels. All the industry configurations studied in this paper (DD, ID/DI, and II) have been shown to have equilibrium gaming theory results. In this paper we can reasonably assume that the industry configurations are exogenous and they focus on the issue of industry efficiency.

To study channel efficiency, Perakis and Roels (2007) adopted a classic newsvendor model in which retail price is fixed. They characterized the efficiency (or loss of efficiency from double marginalization) of different supply chain configurations, such as push or pull inventory positioning, two or more stages, serial or assembly systems, single or multiple competing suppliers, and single or multiple competing retailers. Although the definition of efficiency is the same, our models ignore inventory issues, instead making retail price a decision variable. Furthermore, in the multi-channel scenarios considered in (Perakis and Roels, 2007), there is either a common supplier or a common retailer. In our setup, on the contrary, neither is assumed. Farahat and Perakis (2009) identified lower and upper bounds on the channel efficiency when multi-product firms offering differentiated products engage in price competition. In their work, channel coordination or integration is not considered. In our work, we focus on the scenario where each manufacturer offers only one competing but differentiated product. Another relevant work is (Adida and DeMiguel, 2011). They studied a supply chain configuration where multiple manufacturers supply a set of products to multiple risk-averse retailers who compete in quantities. They have found that the supply chain efficiency (defined the same as industry efficiency in our paper) can be raised to one by inducing the right degree of retailer differentiation. The key differences between their paper and our paper are three-fold. First, we limit our attention to two manufacturers and two retailers who are risk neutral. Second, we consider price competition instead of quantity competition. Third, we also study the impact of vertical integration. It is worth noting that the efficiency can be one in the decentralized industry (DD configuration) under price competition in our study.

The demand asymmetry in our model is analogous to the retailer asymmetry in (Inderst and Shaffer, 2009), in which they considered a model of a monopolistic supplier with two asymmetric retailers. They found that the larger retailer actually obtains a lower wholesale price under the optimal discriminatory two-part tariffs and allocative efficiency also favors the more productive firm. Our model studies the effect of vertical integration in addition to demand asymmetry. Similar setting can be found in (Sorek, 2016), although the latter focused on healthcare market with option demand. A good survey for the competitive effects of vertical integration can be found in (Riordan 2008). For an (almost) symmetric product market model with identical demand functions, one can see, for example, (Loertscher and Reisinger 2014).

III. MODEL

Let q_i represent the demand for product i and p_i denote the retail price of product i. We employ the following demand function:

$$q_i = a_i - p_i + b(p_j - p_i),\ i, j = 1, 2,\ i \neq j, \tag{1}$$

This function is similar to that in (Raju, Sethuraman, and Dhar, 1995) and that in (Indest and Shaffer, 2009). a_i captures the demand potential of product i. Without loss of generality, we assume $a_2 = ka_1$, where k is a scalar between zero and one inclusively. So $a_1 \geq a_2$. This demand asymmetry is either because product 1 is far superior or because channel 1 is more efficient. The degree of demand asymmetry is captured by the value of k. $k = 0$ represents the case in which the demand of product 2 is extremely small compare to that of product 1. We choose the value $k = 0$ instead of a very small number for the ease of computation; there is no essential difference between the case of $k = 0$ and k is a very small number.

Similar to (Inderst and Shaffer, 2009) and (Wu and Mallik, 2010), b is an indicator of the degree of horizontal competition or differentiation, which can be a result of product difference or retail differentiation. The larger b is, the less differentiated the two products/channels are; and thus the horizontal competition is more intense.

Let w_i be the wholesale price of product i. Let y_{M_i} and y_{R_i} denote the respective profits of manufacture i (denoted by M_i) and retailer i (denoted by R_i). The channel profit y_i is the sum of y_{M_i} and y_{R_i}. The industry profit is denoted by $y \equiv y_1 + y_2$. Superscripts are used to denote the industry configuration. For example, y_i^j is the profit of channel i (i = 1 or 2) in industry configuration j, j = DD, DI/ID, II, or B. Asterisk "*" will be used to indicate equilibrium or optimal values.

For any given industry configuration, we use a non-cooperative game to examine the equilibrium prices, quantities, and profits. For the game associated with each industry configuration, the corresponding decision rules and sequences are summarized in the table below.

The equilibrium results of the benchmark case and above games are in Tables 2-5below.

We define industry efficiency as $e^i = y^{i*}/y^{B*}$, where i = DD, DI, ID, and II. This definition of efficiency is consistent with the existing definition in literature (Adida and DeMiguel, 2011). From the results in Tables 2-5, it can be easily verified that e^i is independent of a_1 and a function of only k and b. Later in Section 4, we will examine how vertical integration, differentiation (or horizontal competition) and demand asymmetry affects industry efficiency.

TABLE 1. DECISION RULES AND SEQUENCE IN THE FOUR INDUSTRY CONFIGURATIONS.

Game Stage	DD	DI	ID	II
1	M_1 and M_2 set w_1 and w_2 to maximize $y_{M_i} = w_i q_i$, i = 1, 2.	M_1 sets w_1 to maximize $y_{M_1} = w_1 q_1$.	M_2 sets w_2 to maximize $y_{M_2} = w_2 q_2$.	N/A
2	R_1 and R_2 set p_1 and p_2 to maximize $y_{R_i} = (p_i - w_i) q_i$, i = 1, 2, respectively.	R_1 and channel 2 set p_1 and p_2 to maximize $y_{R_1} = (p_1 - w_1) q_1$, $y_2 = p_2 q_2$.	Channel 1 and R_2 set p_1 and p_2 to maximize: $y_1 = p_1 q_1$, $y_{R_2} = (p_2 - w_2) q_2$.	Two channels set p_1 and p_2 to maximize $y_i = p_i q_i$, i = 1, 2.

TABLE 2. EQUILIBRIUM VALUES IN THE BENCHMARK CASE AND THE II CONFIGURATION.

	Benchmark (B)	Two Vertically Integrated Channels (II)
w_i^*	N/A	N/A
p_i^*	$p_1^{B^*}=\frac{a_1(1+b+kb)}{2(1+2b)},\ p_2^{B^*}=\frac{a_1(k+b+kb)}{2(1+2b)}$	$p_1^{II^*}=\frac{a_1(2+2b+kb)}{4+8b+3b^2},\ p_2^{II^*}=\frac{a_1(2k+b+2kb)}{4+8b+3b^2}$
q_i^*	$q_1^{B^*}=\frac{a_1}{2},\ q_2^{B^*}=\frac{ka_1}{2}$	$q_1^{II^*}=\frac{a_1(1+b)(2+2b+kb)}{4+8b+3b^2},\ q_2^{II^*}=\frac{a_1(1+b)(2k+b+2kb)}{4+8b+3b^2}$
$y_{M_i}^*$	N/A	$y_{M_1}^{II^*}=\frac{a_1^2(1+b)(2+2b+kb)^2}{(4+8b+3b^2)^2}\ y_{M_2}^{II^*}=\frac{a_1^2(1+b)(2k+b+2kb)^2}{(4+8b+3b^2)^2}$
$y_{R_i}^*$	N/A	N/A
y^*	$y^{B^*}=\frac{a_1^2[2kb+(1+b)(1+k^2)]}{4(1+2b)}$	$y^{II^*}=\frac{a_1^2(1+b)[(5b^2+8b+4)(k^2+1)+8kb(1+b)]}{(4+8b+3b^2)^2}$

TABLE 3. EQUILIBRIUM VALUES IN THE DD CONFIGURATION.

	Two Vertically Decentralized Channels (DD)
w_i^*	$w_1^{DD^*}=\frac{a_1[(1+b)(8+16b+5b^2)+2kb(3+6b+2b^2)]}{(4+7b+b^2)(4+9b+3b^2)},\ w_2^{DD^*}=\frac{a_1[k(1+b)(8+16b+5b^2)+2b(3+6b+2b^2)]}{(4+7b+b^2)(4+9b+3b^2)}$
p_i^*	$p_1^{DD^*}=\frac{2a_1(3+6b+2b^2)[(1+b)(8+16b+5b^2)+2kb(3+6b+2b^2)]}{(4+7b+b^2)(4+9b+3b^2)(4+8b+3b^2)}$, $p_2^{DD^*}=\frac{2a_1(3+6b+2b^2)[k(1+b)(8+16b+5b^2)+2b(3+6b+2b^2)]}{(4+7b+b^2)(4+9b+3b^2)(4+8b+3b^2)}$
q_i^*	$q_1^{DD^*}=\frac{a_1(1+b)(2+4b+b^2)[(1+b)(8+16b+5b^2)+2kb(3+6b+2b^2)]}{(4+7b+b^2)(4+9b+3b^2)(4+8b+3b^2)}$, $q_2^{DD^*}=\frac{a_1(1+b)(2+4b+b^2)[k(1+b)(8+16b+5b^2)+2b(3+6b+2b^2)]}{(4+7b+b^2)(4+9b+3b^2)(4+8b+3b^2)}$
$y_{M_i}^*$	$y_{M_1}^{DD*}=\frac{a_1^2(1+b)(2+4b+b^2)[(1+b)(8+16b+5b^2)+2kb(3+6b+2b^2)]^2}{[(4+7b+b^2)(4+9b+3b^2)]^2(4+8b+3b^2)}$, $y_{M_2}^{DD*}=\frac{a_1^2(1+b)(2+4b+b^2)[k(1+b)(8+16b+5b^2)+2b(3+6b+2b^2)]^2}{[(4+7b+b^2)(4+9b+3b^2)]^2(4+8b+3b^2)}$
$y_{R_i}^*$	$y_{R_1}^{DD*}=\frac{a_1^2(1+b)(2+4b+b^2)^2[(1+b)(8+16b+5b^2)+2kb(3+6b+2b^2)]^2}{[(4+7b+b^2)(4+9b+3b^2)]^2(4+8b+3b^2)^2}$, $y_{R_2}^{DD*}=\frac{a_1^2(1+b)(2+4b+b^2)^2[k(1+b)(8+16b+5b^2)+2b(3+6b+2b^2)]^2}{[(4+7b+b^2)(4+9b+3b^2)]^2(4+8b+3b^2)^2}$
y^*	$y^{DD^*}=y_{M_1}^{DD^*}+y_{M_2}^{DD^*}+y_{R_1}^{DD^*}+y_{R_2}^{DD^*}$

TABLE 4. EQUILIBRIUM VALUES IN THE ID CONFIGURATION.

	Integrated Channel 1 and Decentralized Channel 2 (ID)
w_i^*	$w_2^{ID^*}=\frac{a_1(2k+b+2kb)}{2(2+4b+b^2)}$
p_i^*	$p_1^{ID^*}=\frac{a_1[2kb(3+6b+2b^2)+(1+b)(8+16b+5b^2)]}{2(2+4b+b^2)(4+8b+3b^2)}$, $p_2^{ID^*}=\frac{a_1(3+6b+2b^2)(2k+b+2kb)}{(2+4b+b^2)(4+8b+3b^2)}$
q_i^*	$q_1^{ID^*}=\frac{a_1(1+b)[2kb(3+6b+2b^2)+(1+b)(8+16b+5b^2)]}{2(2+4b+b^2)(4+8b+3b^2)}$, $q_2^{ID^*}=\frac{a_1(1+b)(2k+b+2kb)}{2(4+8b+3b^2)}$
$y_{M_i}^*$	$y_1^{ID^*}=\frac{a_1^2(1+b)[2kb(3+6b+2b^2)+(1+b)(8+16b+5b^2)]^2}{4(2+4b+b^2)^2(4+8b+3b^2)^2}$, $y_{M_2}^{ID^*}=\frac{a_1^2(1+b)(2k+b+2kb)^2}{4(2+4b+b^2)(4+8b+3b^2)}$
$y_{R_i}^*$	$y_{R_2}^{ID^*}=\frac{a_1^2(1+b)(2k+b+2kb)^2}{4(4+8b+3b^2)^2}$
y^*	$y^{ID^*}=y_1^{ID^*}+y_{M_2}^{ID^*}+y_{R_2}^{ID^*}$

TABLE 5. EQUILIBRIUM VALUES IN THE DI CONFIGURATION.

	Decentralized Channel 1 and Integrated Channel 2 (DI)
w_i^*	$w_1^{DI^*}=\frac{a_1(2+2b+kb)}{2(2+4b+b^2)}$
p_i^*	$p_1^{DI^*}=\frac{a_1(3+6b+2b^2)(2+2b+kb)}{(2+4b+b^2)(4+8b+3b^2)}$, $p_2^{DI^*}=\frac{a_1[2b(3+6b+2b^2)+k(1+b)(8+16b+5b^2)]}{2(2+4b+b^2)(4+8b+3b^2)}$
q_i^*	$q_1^{DI^*}=\frac{a_1(1+b)(2+2b+kb)}{2(4+8b+3b^2)}$, $q_2^{DI^*}=\frac{a_1(1+b)[2b(3+6b+2b^2)+k(1+b)(8+16b+5b^2)]}{2(2+4b+b^2)(4+8b+3b^2)}$
$y_{M_i}^*$	$y_{M_1}^{DI^*}=\frac{a_1^2(1+b)(2+2b+kb)^2}{4(2+4b+b^2)(4+8b+3b^2)}$, $y_2^{DI^*}=\frac{a_1^2(1+b)[2b(3+6b+2b^2)+k(1+b)(8+16b+5b^2)]^2}{4(2+4b+b^2)^2(4+8b+3b^2)^2}$
$y_{R_i}^*$	$y_{R_1}^{DI^*}=\frac{1}{4}\cdot\frac{a_1^2(1+b)(2+2b+kb)^2}{(4+8b+3b^2)^2}$
y^*	$y^{DI^*}=y_{M_1}^{DI^*}+y_2^{DI^*}+y_{R_1}^{DI^*}$

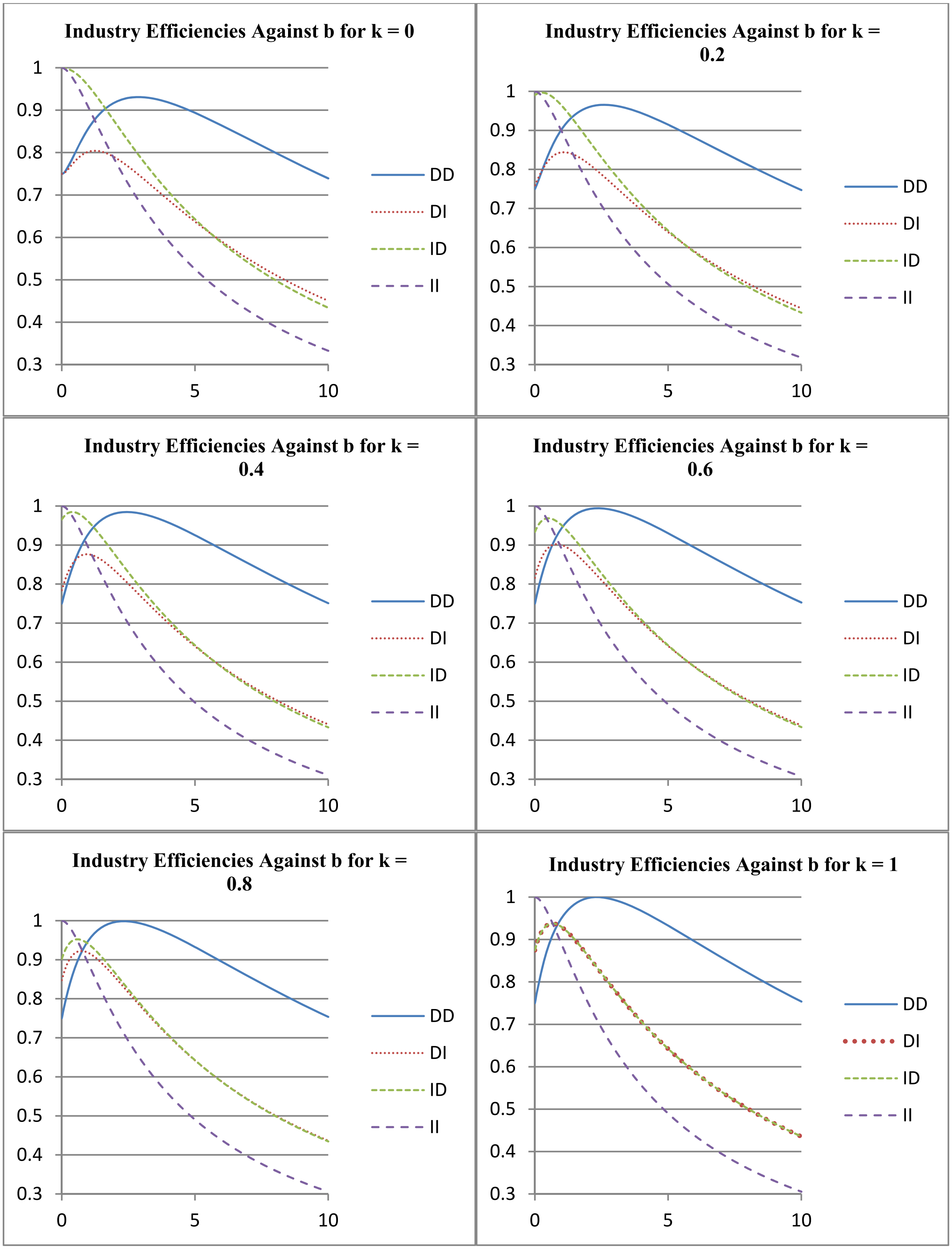

FIGURE 1. EFFICIENCIES OF INDUSTRY CONFIGURATIONS DD, DI, ID AND II.

IV. RESULTS AND ANALYSES

Most results in Propositions 1-5 can be qualitatively observed from Figure 1 below. The official proofs of these findings are provided in the Appendix. Figure 1 depicts the industry efficiencies of the DD, DI, ID and II configurations against the degree of differentiation, b, for given values of k. We choose to show the graph under six specific values of k, 0, 0.2, 0.4, 0.6, 0.8 and 1. It is worth noting that when k = 1, the DI and ID configurations have identical efficiency since they are symmetric with $a_1 = a_2$. Therefore, in the last graph of Figure 1, we can only see three curves, with the efficiency curves of the DI and ID configurations being the same.

Proposition 1.
All the efficiencies, e^{DD}, e^{DI}, e^{ID}, and e^{II} are quasi-concave in b.

Proof. See Appendix.

It is not surprising that the efficiency of II configuration peaks when there is perfect differentiation (or zero substitution between the two products/channels). When b = 0, two channels are essentially monopolist of their respective market. Therefore, vertical integration is the apparent choice for each channel. It is more interesting to see that there exists a unique, positive level of channel differentiation that maximizes industry efficiency in DD, DI and ID configurations. It is possible for supply chains and their members to determine a degree of differentiation (whether it is product differentiation or retail/channel differentiation) such that the industry profitability is maximized.

Let b^j be the value of b that maximizes the efficiency of industry configuration j. That is,

$$b^j = \underset{b}{argmax}\ \ e^j, j = \text{DD, DI/ID, or II.} \tag{7}$$

Proposition 2.
$\frac{db^{DD}}{dk} < 0$, $\frac{db^{DI}}{dk} < 0$, $\frac{db^{ID}}{dk} > 0$, and $b^{II} = 0$ for all k.

Proof. See Appendix.

Proposition 2 states that the optimal level of differentiation decreases in k, in the DD and DI configurations; it increases in k in the ID configuration; and it does not change in k in the II configuration.

Since larger k implies less demand asymmetry, in vertically decentralized configuration as less demand asymmetry presents, less horizontal integration is preferred; in vertically integrated configuration, demand asymmetry has no influence on horizontal integration; in mixed configurations, the effect of demand asymmetry on horizontal integration can be positive or negative depending on whether the decentralized channel has more or less market demand.

Proposition 3.
When k = 1 and b = 2.32, e^{DD} = 1.

Proof. See Appendix.

This is an exciting and important result. Even if all parties involved in an industry with two competing channels pursue the best of their respective interests without any collusion or conspiracy, it is still possible to yield a monopoly-like outcome as equilibrium. The existence of horizontal competition/differentiation and vertical competition plays an essential role in such a finding. Horizontal differentiation prevents a result of marginal cost pricing. Vertical competition further drives retail prices higher.

Proposition 3 has strong implication with respect to anti-trust laws, particularly when both vertical and horizontal competition are present. Monopoly is not always an outcome of mergers andacquisitions, collusion or conspiracy; it can be a natural result of vertical and horizontal competition. In the DD configuration, two competing supply chains can differentiate their products or channels in such a way that the entire industry is like a monopolist.

Next, we compare the efficiencies across the four industry configurations and study which one is the most efficient. To that end, let $b^{j1=j2}$ be the value of b at which $e^{j1} = e^{j2}$, where $j1$ and $j2$ = DD, DI/ID, or II.

Proposition 4.

$e^{II} \geq max(e^{DD}, e^{DI}, e^{ID})$ for $0 \leq b \leq b^{II=ID}$;

$e^{ID} \geq max(e^{DD}, e^{DI}, e^{II})$

for $b^{II=ID} \leq b \leq b^{ID=DD}$;

$e^{DD} \geq max(e^{ID}, e^{DI}, e^{II})$ for $b^{ID=DD} \leq b$.

Proof. See Appendix.

Proposition 4 states the following: (i) when b is sufficiently small, the II configuration is the most efficient configuration among the four; (ii) when b is sufficiently large, the DD configuration is the most efficient; and (iii) when b is of medium range, the ID configuration is the most efficient. Proposition 4 also implies that the DI configuration will never be the most efficient. Therefore, if one channel needs to be integrated, it is better to integrate the more dominant channel from the perspective of the industry.

Conventional wisdom believes that vertical channel integration is always more efficient and beneficial to the channel members involved (Simchi-Levi, Kaminsky, and Simchi-Levi, 2008). Literatures such as (Wu and Mallik, 2010) looked into the issue of vertical integration from the perspective of a single firm or channel and concluded that vertical integration might not always be a better option. Our work, Proposition 4 in particular, looks at this issue from the perspective of the entire industry. According to Proposition 4, there exists an optimal, mutually beneficial (from the standpoint of all members of the industry) industry configuration. Such an optimal configuration is not always in support of vertical integration. A firm must take horizontal competition into account when considering vertical channel integration. More generally speaking, when firmsapply system thinking to supply chain management and consider vertically integrating their channels, they had better expand their system thinking to include a much broader system, the entire industry. The potential strategic integration between competing channels may negate the benefits that are supposed to come with vertical integration.

Proposition 5.

$\frac{db^{II=ID}}{dk} > 0$ and $\frac{db^{DD=ID}}{dk} < 0$.

Proof. See Appendix.

The results in Proposition 5 are visualized in Figure 1. The crossing point between the curves of e^{II} and e^{ID} is shifting towards right as k increases. On the other hand, the crossing point between the curves of e^{ID} and e^{DD} is shifting towards left as k increases. It indicates that the range of b values in which the ID configuration is the most efficient shrinks as k increases. In other words, the ID configuration is less and less likely to be the most efficient configuration as k increases. Hence as the demands of the two channels get closer, the ID configuration is less likely to be the optimal configuration.

V. CONCLUSIONS

We consider an industry with two competing products. Each product is sold exclusively through an either vertically integrated or decentralized channel. Under such a framework, we study how vertical integration/competition, horizontal differentiation, and demand asymmetry would affect industry efficiency.

We have found that monopoly is not necessarily a result of mergers or acquisitions or collusion; it can be an equilibrium outcome of vertical and horizontal competition with all the parties involvedpursue their best respective interests.

We also have found that there exists a unique degree of differentiation that maximizes industry profitability for each of the four industry configurations studied in this paper. Since the degree of vertical differentiation is exogenous rather than a decision variable, our study provides a guideline to choose the best supply chain configuration in terms of efficiency. Furthermore, when looking at a single supply chain, it may be true that vertical integration is always beneficial. However, when this channel is also competing with another channel in the same industry, vertical integration could hurt both the integrated channel and the industry as a whole.

Our work opens doors to future research. As is well known, competition nowadays is competition among supply chains. One key area that warrants further investigation is the study of anti-trust law in a framework with multiple competing supply chains in an industry. Traditionally, anti-trust law aims at regulating business organizations to promote fair competition and to protect consumers, particularly in the events such as mergers and acquisitions, collusions and conspiracies, and monopolies. To the best of our knowledge, however, anti-trust law has never been carefully examined in the presence of both vertical and horizontal competition. Our paper, hopefully, will serve as a starting point for the future research in this fertile area. Another interesting future research will be to investigate marketing strategies to achieve certain specific values of k and b.

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APPENDIX

Proof of Proposition 1

We will only show that e^{II} is quasi-concave; the others can be proved similarly.It is clear that $e^{II}$is a rational function in b, with degrees of the numerator and denominator to be 4 and 5, respectively. Actually, its denominator is $(3b+2)^2(b+2)^2(k^2b+2kb+b+k^2+1)$. Note that the three roots of the denominator are all negative. An easy calculation reveals that the limit of e^{II} is 1 when b approaches 0 from the positive side and is 0 when b approaches infinity.

The numerator of e^{II} is $4(b+1)(2b+1)f(b)$, where$f(b)$ is a quadratic polynomial in b with all positive coefficients. Hence, the roots of the numerator are all negative. The discriminant of $f(b)$ is $-16(k^2-1)^2$, hence $f(b)$ is always positive. Note that $-1/2 < -(1+k^2)/(1+k)^2$, we know that the biggest root of both the denominator and the numerator is $-1/2$. Hence, when the fact $b > 0$ is incorporated into the analysis, we know the function e^{II} will either monotonically increases then decreases to 0 with a horizontal asymptote when b approaches infinity, or monotonically decreases to 0 with the same asymptote. To distinguish them, we notice that the evaluation of e^{II} at 0 is already its maximum value 1, hence it cannot be increasing first. Thus e^{II} is monotonically decreasing when b>0, thus quasi-concave.

Proof of Proposition 2

We will only show that the partial derivative of b^{ID} to k is negative. The cases of b^{DD} and b^{DI} are similar, whereas $b^{II} = 0$ is obvious since e^{II} always reaches maximum at b = 0.

By a similar discussion to the proof of Proposition 1, we know the function e^{ID} will either monotonically increases then decreases to 0 with a horizontal asymptote when b approaches infinity, or monotonically decreases to 0 with the same asymptote. But an easy calculation reveals that the evaluation of e^{ID} at $b = 0$ is slightly less than the evaluation of e^{ID} at $b = 0.1$, hence e^{ID} will monotonically increase then decrease to 0. Thus there exists a unique value b^{ID} that achieves the maximum value of e^{ID}.

The actual value of b^{ID} is equal to the (only real) solution of $(e^{ID})' = 0$. However, solving that equation involves solving a polynomial of degree 9, which is infeasible. Notice that the value of e^{ID} is denominated by the highest term of b (when b is large) and by the constant term (when b is small). Hence we simplify the expression of e^{ID} by eliminating the middle terms of both the denominator and the numerator of it. Then we solve the equation by the first order condition in b, whose solution is the 9th root of $(1 + k^2)/(1 + k)^2$ multiplied by some positive constant. Hence, after an easy calculation, the derivative of b^{ID} to k equals to $k - 1$ multiplied by some positive terms. Thusthe derivative of of$b^{ID}$to k is negative.

Proof of Proposition 3

When k = 1 and b = 2.32, the industry profit of benchmark case is

$$y^{B^*} = \frac{a_1^2[2kb + (1 + b)(1 + k^2)]}{4(1 + 2b)} = 0.5a_1^2$$

and the industry profit of DD configuration is

$$y^{DD^*} = y_{M_1}^{DD^*} + y_{M_2}^{DD^*} + y_{R_1}^{DD^*} + y_{R_2}^{DD^*} = 0.5a_1^2.$$

Hence $e^{DD} = \frac{y^{B^*}}{y^{DD^*}} = 1$.

Proof of Proposition 4

Again, we will show the first part as an example; the other parts can be treated similarly.

It is clear that e^{II} is bigger than all the other three at $b = 0$. It remains to show that of $b^{II=ID}$ is less than both $b^{II=DI}$ and $b^{II=DD}$ to finish the proof. The idea of calculating them are similar so we only show how to calculate $b^{II=ID}$ and $b^{II=DI}$ as an example.

Solving the equation $b^{II=ID}$ leads to a polynomial equation in b of degree 8. By eliminating the middle terms, we get that $b^{II=ID}$ is equal to the 6th root of $16k^2/(12k^2 + 24k + 9)$. Similarly, we find that $b^{II=DI}$ is equal to the 6th root of $16/(9k^2 + 12k)$. By the property $0 < k < 1$, it is easy to compare these two values and it turns out that $b^{II=ID} < b^{II=DI}$, which means that the curve of e^{II} intersects the curve of e^{ID} earlier than it intersects the curve of e^{DI}. A similar comparison $b^{II=ID} < b^{II=DD}$, completes the proof.

Proof of Proposition 5

This is clear by taking derivative of $b^{II=ID}$ and $b^{DD=ID}$ with respect to k.